BRIDGING TRADITION AND INNOVATION: A LITERATURE REVIEW ON PORTFOLIO OPTIMIZATION

Ștefan RUSU, Marcel BOLOȘ

University of Oradea, Oradea, Romania
stefan.rusu.d@gmail.com
marcel_bolos@yahoo.com

Abstract: *Portfolio optimization plays a crucial role in investment decision-making by balancing risk and return objectives. With the aim of improving portfolio performance, while enhancing risk management, this literature review explores traditional and artificial intelligence-powered approaches for portfolio optimization. From the traditional methods of portfolio optimization, methods such as random matrix theory, shrinkage estimators, correlation asymmetries and partial correlation networks are presented. While, from the artificial intelligence realm, techniques such as machine learning efficient frontiers, performance-based regularization, neural network predictors and deep learning models for direct optimization of portfolio Sharpe ratio are highlighted. Intertwining the traditional methods, with artificial intelligence techniques, this review highlights relevant portfolio optimization research useful for academics and practitioners alike.*

Keywords: *artificial intelligence; machine learning; portfolio optimization; finance; investing; financial markets*

JEL Classification: C01; C02; C58; C61; G11; G17

1. Introduction

Portfolio optimization is an important aspect of invesing decision-making. Itrefers to the problem of selection of the best combination of equities in order to build the best portfolio possible given specific constraints, or the selection of the best portfolio out of a pool of considered portfolios. The selection is done through the maximization of specific factors, such as maximizing the expected return or minimizing the undertaken risk, to fit specific investment objectives. Traditional methods of portfolio optimization have relied on statistical and econometric models and methods to measure potential risk and return and to estimate asset correlations and covariance matrices. Focusing on historical data, these methods prove to be inefficient in several situations such as volatile markets and changing market conditions. In the recent years, there has been a growing interest in using artificial intelligence and machine learning techniques to address the aforementioned issues and enhance portfolio optimization.

This literature review explores both the traditional, as well as the artificial intelligence-powered approaches to portfolio optimization to highlight techniques that could help investors in their stock market investment process. By synthetizing insights from various research, the aim of this review is to provide a glimpse in the evolving domain of portfolio optimization and explore the potential benefits of integrating artificial intelligence techniques with traditional methodologies.

2. Literature Review

The work of Laloux et al. (Laloux, et al., 1999) explores the use of random matrix theory (RMT) as a means of comprehending the statistical structure of correlation matrices in financial markets by examining the empirical correlation matrices of the S&P500 stocks and other major markets. The authors then proceed to compare these matrices with theoretical predictions that assume randomness in correlation matrices. The results indicate an agreement between the anticipated theoretical projections and the observed information, specifically with regards to the density of eigenvalues. The research highlights that a significant proportion of the eigenvalues present in the correlation matrices stick to the theoretical formula, thereby suggesting that a small portion of the eigenvectors carry significant information. The discovery implies that the use of random matrix theory can effectively extract relevant correlations among financial assets, thereby facilitating a deeper understanding of the underlying structure of the market. The discovery helps to improve risk management techniques through the detection and measurement of pertinent correlations within complex financial time series data. Using these findings, professionals can improve their risk management strategies and make well-informed decisions in regarding portfolio optimization.

Rebonato and Jäckel (Rebonato & Jäckel, 1999) introduced two methods for generating feasible correlation matrices in situations where the econometric matrix is either noisy, unavailable, or unsuitable. The initial method presents a broader scope in comparison to pre-existing methodologies and provides improved computational efficacy. The system facilitates the generation of a feasible solution that is optimal, considering the criteria specified by the user. The alternative approach, although less general in scope, exhibits remarkable speed and yields outcomes that closely approximate those obtained from the comprehensive methodology. The use of this approach may be either independent or serve as a starting point to facilitate the overall optimization process, thereby enhancing its computational efficiency. The aforementioned methods provide practical solutions for producing viable correlation matrices and make a valuable contribution to the existing research in this field.

Ledoit and Wolf (Ledoit & Wolf, 2003) have introduced a shrinkage estimator that can be used for the purpose of estimating the covariance matrix of stock returns. The use of the shrinkage estimator involves combining the sample covariance matrix with the single-index covariance matrix. The method offers a versatile means of integrating extra-market covariance, without requiring the specification of an arbitrary multifactor structure and displays superior performance compared to existing estimators, such as multifactor models, with respect to the selection of portfolios that have lower out-of-sample variance. The authors emphasize the advantages of accurate estimation of the covariance matrix, not only for portfolio selection, but also for various other applications, including assessments of the Capital Asset Pricing Model (CAPM) and predictive cross-sectional regressions. The adoption of a technique that involves reducing the sample covariance matrix towards the covariance matrix of the single-index model results in the creation of an estimator that is more efficient and well-conditioned. The study demonstrates that the ideal level of shrinkage intensity remains stable across varying sample sizes and proposes a reliable approach for estimating this constant. Empirical evaluations of alternative covariance matrix estimators reveal that the shrinkage method outperforms other methods, particularly in the context of constructing minimum variance stock portfolios using the Markowitz's (Markowitz, 1952) portfolio selection approach. The use of shrinkage estimator is aimed at fulfilling the requirement for an estimator that is both invertible and precise, thereby enhancing the efficacy and comprehensibility of such analyses. The research also advocates for the adoption of generalized least squares (GLS) regressions, which exhibit superiority over ordinary least squares (OLS) regressions due to their association with portfolio selection and recognition of the correlation among stock returns.

Andersen et al. (Andersen, et al., 2001) have examined the volatilities and correlations of daily equity returns that have been realized through analyzing high-frequency intraday transaction prices of individual stocks in the Dow Jones Industrial Average. Their initial observation is that the unconditional distributions of realized variances and covariances show a significant degree of right-skewness. On the other hand, the realized logarithmic standard deviations and correlations demonstrate a distribution that resembles a Gaussian distribution. The Gaussian distributions are also observed in the returns that have been scaled by the realized standard deviations. The second point relates to the strong temporal dependence exhibited by the realized volatilities and correlations, which can be effectively characterized through long-memory processes. Finally, substantial evidence suggests that realized volatilities and correlations move together in a manner consistent with a latent factor structure. Andersen et al. (Andersen, et al., 2001) enhance comprehension of equity return dynamics by emphasizing unique characteristics, including the news impact curve, the asymmetrical relationship between returns and volatility, and the volatility-in-correlation effect. The findings suggest the observed phenomenon can be attributed to a feedback mechanism related to the volatility risk premium, rather than a financial leverage effect. Moreover, the study highlights potential applications of the methodology, such as computing betas to evaluate the validity of the Capital Asset Pricing Model (CAPM), building clear factor models for covariance structure, and simplifying comparisons of volatility predictions produced by different models and techniques. Overall, the research offers significant insights on the intricacies of equity returns and introduces a methodologically simple technique for examining correlations in high-dimensional datasets.

The asymmetries in correlations among U.S. stocks and the aggregate market, with a specific emphasis on disparities between both upward and downward movements were examined by Ang and Chen (Ang & Chen, 2002). Their research presents a new statistic, namely the H statistic, which serves the purpose of comparing and assessing conditional correlation asymmetries. The results indicate that the downside correlations exhibit a considerably greater magnitude than the upside correlations, particularly in instances of extreme downward movements. The correlation asymmetries are distinct from other types of asymmetry measurements, such as skewness and coskewness. The authors have identified multiple factors that are responsible for the asymmetrical nature of correlations. Stocks with small market capitalization, value stocks, and stocks that have previously underperformed the market tend to display more pronounced asymmetric movements. Sectors that are conventionally considered defensive, such as utilities and petroleum, also play a role in the display of correlation asymmetries. The study suggests that stocks with higher beta values, which are deemed to be riskier, demonstrate a lower degree of correlation asymmetry. On the other hand, Ang and Chen’s (Ang & Chen, 2002) research indicates that there is no conclusive relationship between correlation asymmetry and leverage. The research emphasizes that there is an inverse relationship between correlation asymmetries and systematic market risk. The inclusion of correlation asymmetries can prove advantageous in the field of portfolio allocation and risk management strategies.

Kenett et al. (Kenett, et al., 2010) have introduced the concept of partial correlation networks as a means of identifying the dominant stocks that drive correlations within a given stock market. Partial correlation analysis provides a deeper understanding of the correlation structure of a financial market, such as stock returns, by considering the effect of a third variable. The approach enables greater insight into the underlying mechanisms and driving forces that shape the correlation between the two variables. Their findings shed light into the influential role of financial stocks, specifically within the investment services sub-sector, on governing the market's correlation profile. The results underscore the significance of financial stocks in influencing the correlation framework, a function that is not discernible through conventional correlation analysis. The impact of financial stocks is discernible across various levels of stock aggregation, encompassing individual stocks, economic sub-sectors within the financial domain, and the entire sector. The consistency of the influence

of financial stocks over time has been demonstrated through the implementation of moving windows. The utilization of partial correlation analysis in conjunction with conventional correlation analysis is demonstrated as a means of enhancing understanding of the financial market and the underlying factors influencing its correlation structure.
Hartmann et al. (Hartmann, et al., 2001) have investigated the interdependence of asset returns in times of market tension through using an extremal dependence measure. In contrast to conventional correlation analysis, this non-parametric measure has the ability of accommodating non-linear associations and does not depend on presumptions of normal distribution. Their study centers on the use of this metric to examine the relationships among various asset classes during times of market instability. The findings, particularly with regards to the G-5 nations, provide insights regarding the interaction between stock and bond markets in times of market turmoil. The authors indicate that the probability of simultaneous stock market crashes is approximately two times higher than that in bond markets, indicating that stocks are more susceptible to market downturns. Furthermore, the study shows that stock-bond contagion occurs at a similar frequency as flight to quality from stocks into bonds, implying a strong relationship between these asset classes during market instability. The analysis also reveals that extreme cross-border linkages share similarities with national linkages, suggesting that in times of market turbulence, the correlation among assets across international borders corresponds to the correlation among assets within a singular nation. The findings offer investors other aspects to keep in mind when engaging in portfolio construction and optimization.
This study of Junior and Franca (Junior & Franca, 2011) examined the relationship between high market volatility and strong correlations among financial market indexes around the world. By analyzing correlation matrices and eigenvalues of indices during major market turmoil over the past three decades, including events such as Black Monday, the Russian crisis, the burst of the dot-com bubble, and the subprime mortgage crisis, the research shows that financial markets tend to exhibit synchronized behavior during periods of significant downturns. Their study supports earlier results that markets behave similarly during periods of extreme volatility through the investigation of correlation matrices of log-returns for various market indices during financial market crises. However, the study goes beyond a focus only on equities and broadens the analysis to encompass global financial market indices. The authors show that the correlation matrices' eigenvalue probability distributions differ noticeably from those predicted by the Random Matrix Theory, showing the existence of non-random correlations during crises. The study also identifies sets of indices that demonstrate coordinated movements, or "market modes." Additionally, during crises, the probability distributions of the correlation matrices show non-normal behavior with flattened distributions (low kurtosis). The relationship between volatility and average correlation, as determined by Spearman's rank correlation, suggests possible nonlinear relationships between them. A relevant indicator for severe crisis situations is the covariance between average correlation and the volatility of the market mode. The understanding of correlation linkages between financial assets during periods of high market volatility is aided by Junior and Franca’s (Junior & Franca, 2011) findings.
Econometric indicators for assessing systemic risk, which can effectively account for the interconnectedness among various financial entities such as hedge funds, banks, brokers, and insurance companies were studied by Billio et al. (Billio, et al., 2012) by using principal components analysis and Granger-causality tests. The findings indicate a notable increase in the interconnectedness among the four sectors during the previous decade, emphasizing the intricate and ever-changing web of connections that contribute to systemic risk. The complex structure of the system leads to a rise in interconnectivity and systemic vulnerability. The findings indicate that principal components analysis and Granger-causality networks are efficient in detecting market disruption and adversity, exhibiting promising out-of-sample characteristics. Furthermore, the metrics formulated in the research exhibit promising forecasting abilities for economic downturns, in which hedge

funds are recognized as forward indicators of market disruption. The study underscores the significance of taking into consideration feedback effects, risk management practices, and the topology of the financial network in understanding systemic risk. Billio et al. (Billio, et al., 2012) underscore the importance of quantifying, observing, and predicting financial crises, and advocates for the adoption of various methodologies to evaluate the structure of the financial system in order to recognize potential risks and reduce their disruptive outcomes.
Leaving the traditional domain of portfolio optimization techniques, the upcoming scientific papers focus on the use of artificial intelligence, machine learning and deep learning techniques to improve results and efficiency.
An innovative portfolio optimization model that integrates short-term investment opportunities through predictive analysis was proposed by Freitas et al. (Freitas, et al., 2009). A neural network predictor for stock returns was augmented using a risk measure is derived based on the prediction errors. The proposed model leverages efficient diversification through the discovery of predictors with low and complementary pairwise error profiles. Although the concept of using prediction for portfolio selection has been around for a while, traditional approaches have predominantly concentrated on determining the return series' distributions. The efficiency of a neural network predictor is dependent upon the network's topology, training techniques, and the characteristics of the data. The primary argument of Freitas et al. (Freitas, et al., 2009) was that while individual predictors may not demonstrate robust performance, their collective utilization across multiple stocks may yield superior outcomes. The empirical results indicate that portfolios based on prediction are better suited for short-term investment in comparison to portfolios based on mean-variance. The concluding remarks highlight the model's capacity to tackle the issue of varying predictor performances by computing risk metrics based on the individual and collective predictor performances, using the variances and covariances of their prediction errors. Additionally, it underscores the optimality of the model in the selection of efficient portfolios during portfolio transitions, thereby ensuring the generation of superior predicted portfolios.
Ban et al. (Ban, et al., 2016) have studied the limitations of traditional portfolio optimization models when applied to real-world scenarios and recommended the use of machine learning methodologies, such as regularization and cross-validation, to tackle these challenges related to estimation. Ban et al. (Ban, et al., 2016) proposed the use of performance-based regularization (PBR) in addressing mean-variance and mean-conditional value-at-risk (CVaR) problems. The approach involves the application of constraints on sample variances to mitigate the impact of estimation errors. Mean-variance problems are addressed using approximations that rely on rank-1 and convex quadratic techniques. On the other hand, the PBR model is demonstrated to possess a rigorous convex relaxation for mean-CVaR problems. The PBR models are structured as optimization problems that incorporate robustness, using innovative uncertainty sets. The study shows the asymptotic optimality of both sample average approximation (SAA) and PBR solutions. Additionally. empirical investigations using Fama-French data sets are used to compare these techniques to other regularization methods and equally weighted portfolios. The results indicate that PBR shows superior performance compared to alternative benchmarks in 66.7% of the analyzed data sets. Future research of these is recommended to increase their applicability to other decision problems.
Clak et al. (Clark, et al., 2020) presented an approach that integrated portfolio theory and machine learning. Their approach introduced an out-of-sample machine learning efficient frontier, with a particular emphasis on a two-asset portfolio characterized by high and low risk. The authors delve into the economic value of machine learning in contrast to the mean-variance paradigm for portfolio selection, while considering the potential effect of estimation risk. Despite the potential absence of normative appeal in the mean-variance model, the use of the machine learning approach can effectively leverage return predictability and downside risk. The results suggest that the machine learning efficient frontier, exhibits

superior performance compared to the traditional mean-variance efficient frontier during out-of-sample testing. This underscored the possible use of machine learning methodologies in the process of portfolio selection, particularly in the context of evaluating estimation risk. According to the authors, the machine learning efficient frontier exhibits superior performance during times of market instability by reallocating investments from high-risk assets to safe-haven assets, thereby demonstrating its adaptability to dynamic market conditions. Nevertheless, the research exhibits inconsistent findings regarding the effectiveness of the suggested machine learning portfolio strategy in comparison to the naive portfolio approach. Further areas of research include expanding the feature space to encompass a wider range of information, assessing various machine learning algorithms to determine the optimal ones for portfolio selection, and analyzing the influence of a larger number of assets on the effectiveness of the machine learning technique.
Zhang et al. (Zhang, et al., 2020) focused on the use of deep learning models for the optimization of portfolio's Sharpe ratio in a direct manner, thus eliminating the necessity of forecasting expected returns. Their study focused on the use of Exchange Traded Funds (ETFs) in trading market indices for the purpose of constructing a portfolio that leverages the strong correlations that exist among various asset classes. The methodology involves the comparison of the deep learning approach with a diverse array of algorithms. A sensitivity analysis was conducted to evaluate the significance of input variables, while the performance of the method is evaluated across varying cost rates and risk levels using volatility scaling. The successful implementation of deep learning models in the direct optimization of the Sharpe ratio of a portfolio is highlighted. The proposed methodology exhibits superior performance compared with various popular algorithms and techniques, such as those founded on reallocation, mean-variance optimization, maximum diversification, and stochastic portfolio theory. Furthermore, the evaluation of performance includes an important period that encompasses the COVID-19 crash, thereby showcasing the validity and feasibility of the suggested approach.
A novel machine learning framework that combines portfolio optimization and is aware of trading costs is proposed by Jensen et al. (Jensen, et al., 2022). The study employs a methodology that combines trading-cost-aware portfolio optimization with machine learning techniques. The framework developed involves a direct learning method for calculating portfolio weights, which is guided by an economic objective. The authors present the notion of an implementable efficient frontier that incorporates transaction expenses and showcase its superior net-of-cost performance while addressing the limitations of prior methodologies that disregard trading expenses. The empirical results indicate that the proposed method has the ability to widen the implementable efficient frontier in comparison to alternative approaches for portfolio selection. The framework exhibits superiority over sophisticated alternatives, as shown by significant out-of-sample gains and demonstrates the way the implementable efficient frontier applies to investors with greater size who encounter higher market impact costs. Thus, underscoring the real-world implications of incorporating trading expenses in the process of portfolio optimization. The concluding remarks underscore the innovative approach of the suggested technique, which prioritizes the identification of lasting economic features over temporary ones that may appear promising in theory but lack practical efficacy. This underscores the significance of considering the economic significance of securities when constructing a portfolio.

3. Conclussion

To conclude, the paper highlights various traditional and artificial intelligence-powered approaches to portfolio optimization. The traditional methods part starts with random matrix theory for better understanding correlation structures and using covariance matrix estimation for insights into portfolio selection and risk management and continues with research on correlation assymetry, partial correlation networks and extremal dependence measures as used to obtain a deeper understanding of interdependencies among financial assets and market dynamics.Highlighting artificial intelligence, machine learning and deep learning techniques, the second part of the literature review focuses on newer, more innovative research that has the potential to improve several facets of portfolio optimization. Promising results are shown in improving portfolio performance, especially in times of higher volatility, are shown by models integrating machine learning efficient frontiers, neural network predictors or performance-based regulation. While the challenges in traditional portfolio optimization methods are tackled by deep learning models for direct optimization of portfolio Sharpe ratio, or machine learning techniques that take trading costs into account as well. Overall, the integration of artificial intelligence techniques with traditional portfolio optimization problems presents exciting opportunities for researchers and practitioners to improve portfolio returns, while enhance their risk mitigation strategies.

References

1. Andersen, T. G., Bollerslev, T., Diebold, F. X. & Ebens, H., 2001. The distribution of realized stock return volatility. Journal of Financial Economics Volume 61, Issue 1, pp. 43-76.
2. Ang, A. & Chen, J., 2002. Asymmetric correlations of equity portfolios. Journal of Financial Economics Volume 63, Issue 3, pp. 443-494.
3. Ban, G.-Y., El Karoui, N. & Lim, A. E. B., 2016. Machine Learning and Portfolio Optimization. Management Science.
4. Billio, M., Getmansky, M., Lo, A. W. & Pelizzon, L., 2012. Econometric Measures of Systemic Risk in the Finance and Insurance Sectors. Journal of Financial Economics, 104(3), pp. 535-559.
5. Clark, B., Feinstein, Z. & Simaan, M., 2020. A machine learning efficient frontier. Operations Research Letters, 48(5), p. 630–634.
6. Freitas, F. D., Souza, A. F. D. & Almeida, A. R. d., 2009. Prediction-based portfolio optimization model using neural networks. Neurocomputing 72(10-12), p. 2155–2170.
7. Hartmann, P., Straetmans, S. & Vries, C. G. D., 2001. Asset market Linkages in Crisis Periods. European Central Bank Working Paper No. 71.
8. Jensen, T. I., Kelly, B. T., Malamud, S. & Pedersen, L. H., 2022. Machine Learning and the Implementable Efficient Frontier. Swiss Finance Institute Research Paper No. 22-63.
9. Junior, L. S. & Franca, I. D. P., 2011. Correlation of financial markets in times of crisis. Insper, Instituto de Ensino e Pesquisa.
10. Kenett, D. Y. et al., 2010. Dominating Clasp of the Financial Sector Revealed by Partial Correlation Analysis of the Stock Market. PLoS ONE, 5(12), e15032.
11. Laloux, L., Cizeau, P., Bouchaud, J. P. & Potters, M., 1999. Random Matrix Theory and Financial Correlations. International Journal of Theoretical and Applied Finance, 3(03), pp. 391-397.
12. Ledoit, O. & Wolf, M., 2003. Improved estimation of the covariance matrix of stock returns with an application to portfolio selection. Journal of Empirical Finance Volume 10, Issue 5, pp. 603-621.
13. Markowitz, H., 1952. Portfolio Selection. The Journal of Finance, Vol. 7, No. 1, pp. 77-91.
14. Rebonato, R. & Jäckel, P., 1999. The Most General Methodology to Create a Valid Correlation Matrix for Risk Management and Option Pricing Purposes. Journal of Risk, 2, pp. 17-28.
15. Zhang, Z., Zohren, S. & Roberts, S., 2020. Deep Learning for Portfolio Optimization. The Journal of Financial Data Science Fall 2020, 2 (4), pp. 8-20.

Legal aspects

has been granted when submitting their papers. Any material received without such evidence will be assumed to originate from the authors. The author/s will be asked to submit *The Copyright Statement* upon acceptance of the paper.