HETEROSCEDASTICITY CORRECTION MEASURES IN STOCHASTIC FRONTIER ANALYSIS

Rauf I. RAUF[1], Bello A. HAMIDU[2], Bodunwa O. KIKELOMO[3], Ayinde KAYODE[4], Alabi O. OLUSEGUN[5]

[1]Department of Statistics, Federal University of Technology, Akure. ORCID: https://orcid.org/0000-0003-3192-6677.
[2]Department of Statistics, Federal University of Technology, Akure. ORCID: https://orcid.org/0000-0003-1169-5285
[3]Department of Statistics, Federal University of Technology, Akure. ORCID: https://orcid.org/0000-0003-3533-8346
[4]Department of Mathematical Science, Northwest Missouri State University. ORCID: https://orcid.org/0000-0001-6194-6199.
[5]Department of Statistics, Federal University of Technology, Akure. ORCID: https://orcid.org/0000-0002-7218-3251
rauf.ibrahim@outlook.com
habello@futa.edu.ng
okbodunwa@futa.edu.ng
ayindek@nwmissouri.edu
alabioo@futa.edu.ng

Abstract: *The stochastic frontier analysis (SFA) model, designed to assess technical efficiency in production models, operates under the assumption of homoscedasticity. However, in practical scenarios, either the random error or the technical efficiency error, or both, can exhibit non-homoscedasticity. This research proposes heteroscedasticity correction measures for the random error (HCRE), technical efficiency error (HCTE), and both (HCRTE) within the SFA model. The study aims to determine which correction measure yields the most efficient parameter estimates when heteroskedasticity is present. The comparison involves evaluating the mean squared error (MSE) across different forms of heteroscedasticity and sample sizes through Monte Carlo simulations comprising 5000 replications. The findings indicate that attempting to correct for heteroskedasticity in the absence of such issues can adversely affect the parameter estimates of the SFA Model. Conversely, the HCRTE measure consistently produces the most efficient estimates when dealing with heteroskedasticity in terms of both random error and technical efficiency. Moreover, in cases where heteroskedasticity exists, applying the HCRTE measure not only enhances parameter estimates but also improves the technical efficiency measure of the SFA model.*

Keywords: Stochastic Frontier Production; Heteroscedasticity Correction; Technical Efficiency, Monte Carlo Simulation.

Introduction

Stochastic frontier analysis (SFA) emerged significantly with seminal papers by Meeusen and van den Broeck (1977) and Aigner, Lovell, and Schmidt (1977), becoming a promising area of research for scholars. The core objective of SFA is to gauge firm efficiency, with technical efficiency defined by Koopmans (1951) and cited by Battese and Coelli (1977) as the inability to increase one output without decreasing another or using more input. Consequently, SFA measures technical efficiency as the ratio of actual output to potential output. However, the SFA model assumes no heteroscedasticity, which can distort technical efficiency measurements, underscoring the criticality of accurate model performance (Hadri, 1999). Common issues such as multicollinearity, heteroskedasticity, and autocorrelation in least squares regression-based estimation of frontier functions have been overlooked in SFA applications, prompting a focus on developing heteroskedasticity-corrected measures for more efficient SFA estimates. This study aims to propose and compare such measures to enhance the accuracy of technical efficiency estimation within the SFA framework.

Review of the Related Literature

Studies focusing on stochastic frontier analysis (SFA) often encounter issues related to heteroskedasticity, especially when dealing with data from diverse firms varying in size and variables. Historically, economists have linked heteroskedasticity in cross-sectional data to size-related characteristics of observed firms, highlighting the serious implications for least-squares estimation. Jon and Stephen (1995) introduced a method to address heteroskedasticity in cost-frontier estimation, demonstrating significant changes in estimated frontiers and inefficiency measures. In contrast, Taeyoon and K (2008) explored heteroscedasticity in the stochastic frontier cost function using aggregated data and Monte Carlo studies. Their findings suggested that heteroscedasticity predominantly affects the random effect and unexplained error term, affecting the estimation of inefficiency measures. Ford (1999) developed a test for heteroskedasticity in stochastic frontier models, focusing on the error term related to inefficiency measurement and proposing a Lagrange multiplier test akin to Breusch-Pagan's test for linear models.

Further investigations by Caudill and Ford (1993) using Monte Carlo experiments revealed the overestimation of intercepts and underestimation of slope coefficients in the presence of heteroskedasticity. They emphasized the sensitivity of inefficiency measures to specification errors in frontier models. Hadri (1999) expanded on heteroskedasticity corrections in stochastic frontier models, affecting firm-specific inefficiency measures and rankings. In the context of energy demand functions, Llorca et al. (2017) applied SFA to estimate energy efficiency measures at the country level, proposing a random parameter model to accommodate unobserved heterogeneity among countries. Their approach provided robust alternatives for international comparisons, especially in regions like Latin America and the Caribbean with significant energy consumption in the transport sector. Despite these insights, recent and comprehensive studies on heteroskedasticity corrections in SFA models remain limited, highlighting the need for further research to address this gap effectively.

Methodology

3.1 SFA estimation

Following (Battese & Coelli, 1995) as sited by (Nguyen, 2010), the stochastic frontier model used in the study is thus: considering cross-sectional data on the quantities of N inputs x_{ni}, $n = 1, \ldots, N; i = 1, \ldots, I$ are used to produce a single output y_i, $i = 1, \ldots, I$ am available to each of the I producers. The stochastic production frontier for producers can be written as follows.

$$y_i = f(x_i;\beta).\exp(V_i).TE_i$$

(3.1)

Here, the βs are the parameters in the production function. V_i reflects random noise, and TE_i is the output-oriented technical efficiency of producer i. From Equation (3.1) we have;

$$TE_i = \frac{y_i}{f(x_i;\beta).\exp(V_i)}$$

(3.2)

Assuming the $f(x_i;\beta)$ takes a Cobb-Douglas form, the (3.2) becomes

$$TE_i = \exp\{-U_i\}$$

(3.3)

Thus, the stochastic production frontier becomes;

$$\ln y_i = \beta_0 + \sum_{n=1}^{N} \beta_n \ln x_{ni} + V_i - U_i$$

(3.4)

Then the estimate of the technical efficiency can be obtained from: (3.3) and (3.4) (Battese & Coelli, 1977) as sited by (Nguyen, 2010)

$$\widehat{TE_{1i}} = \exp\{-E\widehat{(U_i|E_i)}\}$$

(3.5)

$$\widehat{TE_{2i}} = E(\exp\widehat{\{-U_i\}}|E_i)$$

(3.6)

Here, the joint density of U and V is then given as follows (Nguyen, 2010):

$$f(u,v) = \frac{1}{\sqrt{2\pi}\sigma\theta} \exp\left\{-\frac{v^2}{2\sigma^2}\right\}$$

(3.7)

Since E + U, the joint density of U and E after variable transformation is (Nguyen, 2010):

$$f_{U,E}(u,\epsilon) = \frac{1}{\sqrt{2\pi}\sigma\theta} \exp\left\{-\frac{(\epsilon+u)^2}{2\sigma^2}\right\}$$

(3.8)

Hence, the marginal density of E can be derived (Nguyen, 2010):

$$f_E(\epsilon) = \int_0^{\theta} \frac{1}{\sqrt{2\pi}\sigma\theta} \exp\left\{-\frac{(\epsilon+u)^2}{2\sigma^2}\right\} \mathrm{d}u$$

(3.9)

$$= \int_{\frac{\epsilon}{\sigma}}^{\frac{\theta+\epsilon}{\sigma}} \frac{1}{\sqrt{2\pi}\theta} \exp\left\{-\frac{z^2}{2}\right\} \mathrm{d}z$$

(3.10)

$$= \frac{1}{\theta}\left[\Phi\left(\frac{\theta+\epsilon}{\sigma}\right) - \Phi\left(\frac{\epsilon}{\sigma}\right)\right], \epsilon \in \Re$$

(3.11)

Noting that $F_E(\varepsilon)$ is a symmetric density with a mean of (Nguyen, 2010):

$$E(\epsilon) = -E(u) = -\frac{\theta}{2},$$

and variance of (Nguyen, 2010):

$$Var(\epsilon) = Var(v) + Var(u) = \sigma^2 + \frac{\theta^2}{12}$$

3.2 Proposed Measures for Heteroscedasticity Correction in SFA

After a successful estimation of the SFA model parameters and assuming the decomposed error terms in SFA model which is an established modified ordinary least squares regression model, the assumption of homoscedasticity and thus heteroscedasticity (non-constant variance in the error term) is violated.

This study then proposed the following measures in an attempt to obtain the most efficient measure of heteroskedasticity correction in any given stochastic frontier analysis model considering its nature of the decomposed error terms $(V_i - TE_i)$. Recall from (3.1) a typical stochastic production frontier model that is presented in (3.12) and (3.13) as the log linear and linear functions respectively by:

$$y_{i=f(X_i;\beta).\exp(V_i).TE_i} \quad (3.12)$$

$$y_{i=\beta_0+\beta_1x_1+\beta_2x_2+\cdots+(V_i-TE_i)} \quad (3.13)$$

Then, the proposed measures are:

$$y_{i=\beta_0+\beta_1x_1+\beta_2x_2+\cdots+(\Delta V_i-TE_i)} \quad (3.14)$$

$$y_{i=\beta_0+\beta_1x_1+\beta_2x_2+\cdots+(V_i-\Delta TE_i)} \quad (3.15)$$

$$y_{i=\beta_0+\beta_1x_1+\beta_2x_2+\cdots+\Delta(V_i-TE_i)} \quad (3.16)$$

Where:

(ΔV_i) is when the heteroscedasticity is corrected only in the random error component,
(ΔTE_i) is when the heteroscedasticity is corrected only in the technical efficiency component, while, $(\Delta V_i - TE_i)$ is when heteroscedasticity is corrected in both the random error component and the technical efficiency component of the error term of the SFA model.

In practice, the nature of heteroskedasticity in real-life data is never known. Hence, the error variance is often estimated using a functional form of the residuals obtained from the OLS estimator (White, 1980). Basically, WLS involves the use of weights to correct for the structure of heteroscedasticity present in the model. In the literature, White (1980) is the most widely accepted approach towards generating weights. This procedure is as follows:

From (3.13) the stochastic frontier model with the assumption of heteroskedasticity is given by:

$$y_{i=\beta_0+\beta_1x_1+\beta_2x_2+\cdots+(V_i-TE_i)} \quad (3.17)$$

Letting $\mu_i = (V_i - TE_i)$
Where $\mu_i$is the error term in OLS model.
$y_{i=\beta_0+\ \beta_1 x_1+\ \beta_2 x_2+\cdots+\mu_t}$
(3.18)
Var(μ_i) = E(μ_i^2) = σ^2 for i = 1, 2, …, n

Note that the *t* subscript attached to the sigma square indicates that the disturbance for each of the *n units* is drawn from a probability distribution that has a different variance (Lee, 1983).
Given such a non-constant variance function

$$Var(\,e_i) = \sigma^2 = \sigma^2 x_i^{\alpha}$$

(3.19)

where α is the unknown parameter in the model;
Taking the natural logarithm

$$\ln \sigma^2 = \ln \sigma^2 + \alpha \ln x_i$$

(3.20)

Then take the exponential of the equation.

$$\sigma_i^2 = exp[ln(\sigma_o^2) + \alpha\, ln(x_i)]$$

(3.21)

Letting $\beta_1 = ln(\sigma_i^2)$, $\beta_2 = \alpha$, $Z_i = ln(x_i)$

$$\sigma_i^2 = exp[\beta_1 + \beta_2 Z_i]$$

(3.22)

From equation 3.22, if the variance depends on more than one explanatory variable or from a multiple regression case, equation 3.22 is transformed into equation 3.22* below:

$$\sigma_i^2 = exp[\beta_1 + \beta_2 Z_{i2}+\ldots+\beta_3 Z_{is}]$$

(3.22*)

Thus, we take the exponential function since it gives a non-negative value of variance σ_i^2 .

Therefore, the equation (3.21) becomes $\beta_1 = ln(\sigma_i^2)$,$\beta_2 = \alpha$, $Z_i = ln(x_i)$in multiple regression case. Then, using the OLS technique to estimate the coefficients$\beta_1,\ \beta_2, \ldots, \beta_s$ of the variance function

$$ln(\sigma_i^2) = \beta_1 + \beta_2 Z_{i2} + \cdots + \beta_s Z_{is}$$

(3.23)

Where$Z_{i2} = ln(x_2)$, $Z_{i3} = ln(x_3)$ and so on.
We then took the square root of the exponential of the fitted estimate.

$$\hat{\sigma}_i = \sqrt{exp(\,\hat{\beta}_1 + \hat{\beta}_2 Z_{i2}+\ldots+\hat{\beta}_s Z_{is})}$$

(3.24)

Then $\hat{\sigma}_i$is the weight required to transform the data set by dividing through.
Since;

$$Var\left(\frac{e_i}{\sigma_i}\right) = \frac{1}{\sigma_i^2} Var(e_i) = \frac{1}{\sigma_i^2} \times \sigma_i^2 = 1$$

(3.25)

Using the estimate of our variance function $\hat{\sigma}_i^2$in place of σ_i^2in the equation (3.12) above to obtain the Generalized Least Square Estimator of$\beta_1,\ \beta_2, \ldots, \beta_s$.
We then defined the transformed variables as homoskedasticity.

$$y_i^* = \frac{y_i}{\hat{\sigma}_i},\ x_{i1}^* = \frac{1}{\hat{\sigma}_i},\ x_{i2}^* = \frac{x_i}{\hat{\sigma}_i}, \ldots,\ x_{is}^* = \frac{x_s}{\hat{\sigma}_i}$$
(3.26)

3.3 The Monte Carlo Design

To investigate the finite sample properties of the maximum likelihood (ML) estimators of the half-normal stochastic frontier production functions in the presence of heteroskedasticity, we use a Monte Carlo experiment. We simulate a frontier model using a simple Cobb-Douglass production function of the form:

$$\begin{aligned} \ln y_i &= \alpha + \boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_i + v_i - u_i \\ &= \alpha + \boldsymbol{\beta}^{\mathrm{T}}\mathbf{x}_i + \varepsilon_i. \end{aligned}$$

where v and u are a normal error variable and a half normal error, respectively.
Following (Guermat & Hadri, 1999), we then simulate the independent variables X_i as follows:
Xi are sets of independent variables that are fixed following $X_1 = 1\ldots$ N, $X_2 = 1\ldots \sqrt{N}$ and Z $=\sqrt{i}$, for i = 1… N. We also generate the two error terms as follows $v \sim N(0,\sigma^2)$ and $u \sim |N(0,\sigma^2)|$
Then the model for the heteroskedasticity function was formed as follows:

$$\sigma_v = exp(\alpha_0 + \sigma_{\alpha 1} \ln X_{1i} + \sigma_{\alpha 2} \ln X_{2i})$$
$$\sigma_u = exp\left(\gamma_0 + \sigma_{\gamma 1} \ln Z_i\right)$$

The parameters were then set at:

$$\beta_0 = \alpha_0 = \gamma_0 = \alpha_1 = \alpha_2 = \gamma_1 = 1$$
$$\beta_1 = \beta_2 = 0.5(constantreturntoscale)$$

The parameter σ measures the degree of heteroscedasticity. We use several degrees of heteroscedasticity by letting σ to vary (0, 0.2, 0.3 and 0.4). When σ = 0 we obtain the homoscedastic case. We also considered different sample sizes: 20, 40, 100, 250, and 500 observations (Hadri, Guermat, & Whittaker, 1999). To analyze the effect of heteroscedasticity, we estimated the following models (OBC, OCRE, OCTE, OCEP, SBC, HCRE, HCTE, HCRTE) which represents OBC before any form of correction; OCRE corrected for heteroscedasticity in V; OCTE corrected for heteroscedasticity in W; OCEP corrected for heteroscedasticity empirically; SBC before any form of correction; HCRE corrected for heteroscedasticity in V; HCRE corrected for heteroscedasticity in W; HCRTE corrected for heteroscedasticity empirically, respectively) using the MSE. We also set the number of replications to 5000.

Results

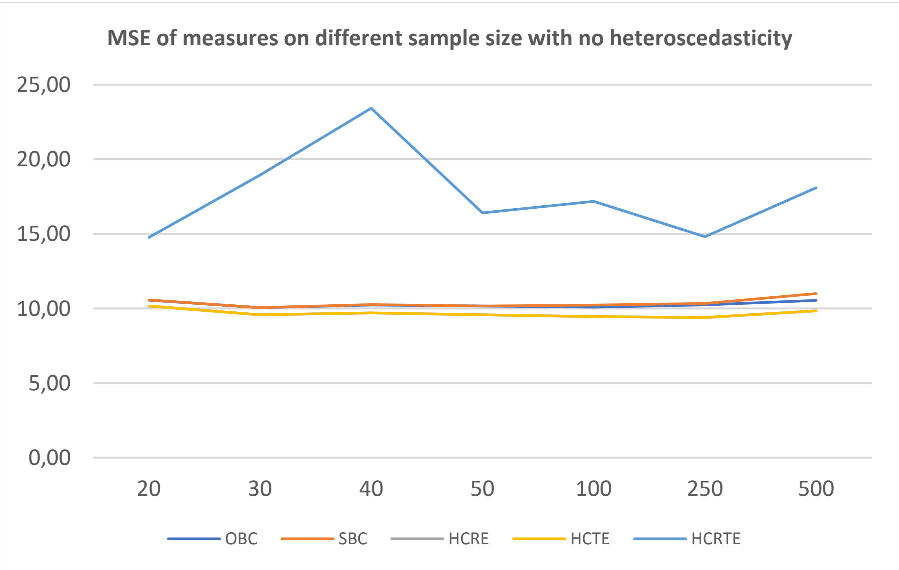
The results from the simulation study for scenarios with no heteroscedasticity and in the presence of heteroscedasticity. The performance of the proposed measures was studied with the behavior of their respective mean square error (MSE).

Table 4.1 MSE of the measures without heteroskedasticity

N	Ordinary Least Square Models				Stochastic Frontier Analysis Models			
	OBC	**OCRE**	**OCTE**	**OCEP**	**SBC**	**HCRE**	**HCTE**	**HCEP**
20	38391.44	38391.77	38391.77	38488.18	39230.07	25938.77	25938.77	2568879.00
30	23307.88	23307.90	23307.90	23308.38	23228.77	14357.29	14357.29	167731082.00
40	27651.82	27651.84	27651.84	27651.85	28518.75	16336.75	16336.75	14815393453.00
50	25897.93	25906.55	25906.55	25899.33	25894.39	14468.46	14468.46	13277272.00
100	24008.15	24008.28	24008.28	24008.47	27665.21	12807.45	12807.45	28951440.00
250	27971.75	27971.83	27971.83	27989.21	30726.15	12000.69	12000.69	2689264.00
500	37934.92	37935.06	37935.06	37941.24	59551.22	18780.36	18780.36	71964505.00

**Estimators with minimum MSE*

Figure 4.1



**The measure with the line closest to zero is the best*

From Table 4.1 and Figure 4.1, the following was observed: The OBC model exhibited the lowest MSE across sample sizes of 20, 30, 40, 50, 100, 250, and 500. This indicates that attempting any form of heteroskedasticity correction, as seen in OCRE, OCTE, and OCEP, would lead to adverse outcomes, as evidenced by the MSE simulation results presented in the table above, due to the absence of heteroskedasticity in the data. Furthermore, it was observed that as the sample size increased (specifically, from 50 to 500), the MSE values for the OBC, OCRE, OCTE, and OCEP models remained relatively consistent. This observation suggests that the proposed correction methods in the OCRE, OCTE, and OCEP models may be more effective with smaller sample sizes. In terms of SFA estimators, both HCRE and HCTE demonstrated minimal MSE values, with identical MSE values across all sample sizes considered. However, the HCRTE model yielded higher MSE values in varying sample sizes.

Table 4.2 MSE of the Proposed Measures with a Varying Level of Heteroskedasticity

N	hetero (sigma)	Stochastic Frontier Analysis Correction Measures			
		SBC	**HCRE**	**HCTE**	**HCRTE**
20	0.1	545.82	487.34	353.44	18.33*
	0.3	330.24	1388.67	311.36	2.42*
	0.5	327.21	8404.04	750.18	1.78*
30	0.1	298.79	236.64	173.55	7.84*
	0.3	165.89	554.37	148.22	0.99*
	0.5	164.83	4271.25	218.29	0.82*
40	0.1	436.44	349.22	247.8	6.86*
	0.3	325.55	1060.7	277.78	1.27*
	0.5	322.34	8564.54	410.01	0.87*
50	0.1	180.15	142.07	99.26	4.45*
	0.3	148.82	484.18	120.76	0.72*
	0.5	147.75	4061.62	183.04	0.75*
100	0.1	48.58	34.81	24.29	1.47*
	0.3	36.7	116.42	28.02	0.80*
	0.5	36.83	1105.16	42.8	0.84*
250	0.1	5.36	3.84	2.75	0.65*
	0.3	3.58	11.13	2.91	0.97*
	0.5	3.58	120.74	4.14	1.00*
500	0.1	1.31	1.29	1.14	0.72*
	0.3	1.45	2.76	1.43	1.05*
	0.5	1.45	24.53	1.61	1.06*

*Estimators with minimum MSE

Figure 4.2 MSE Line Graph for the Proposed Measures at a heteroskedasticity level of 0.1

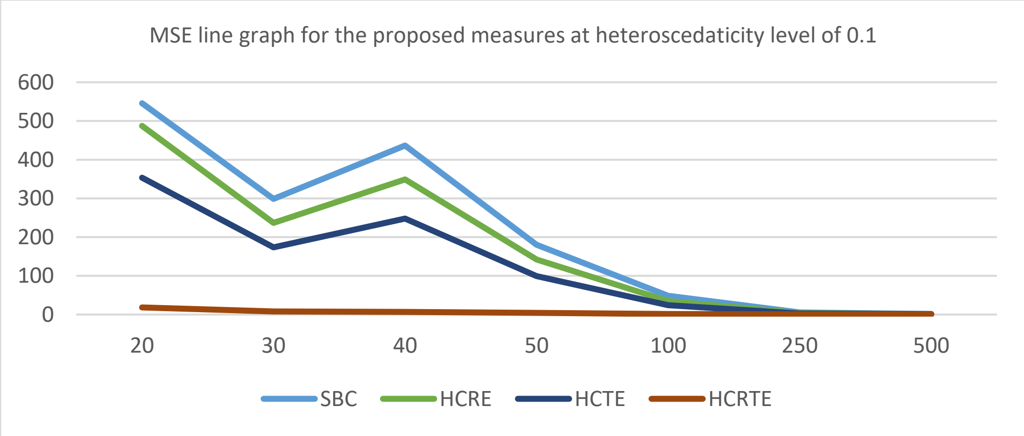


Figure 4.3 MSE line graph for the proposed measures at a heteroskedasticity level of 0.3

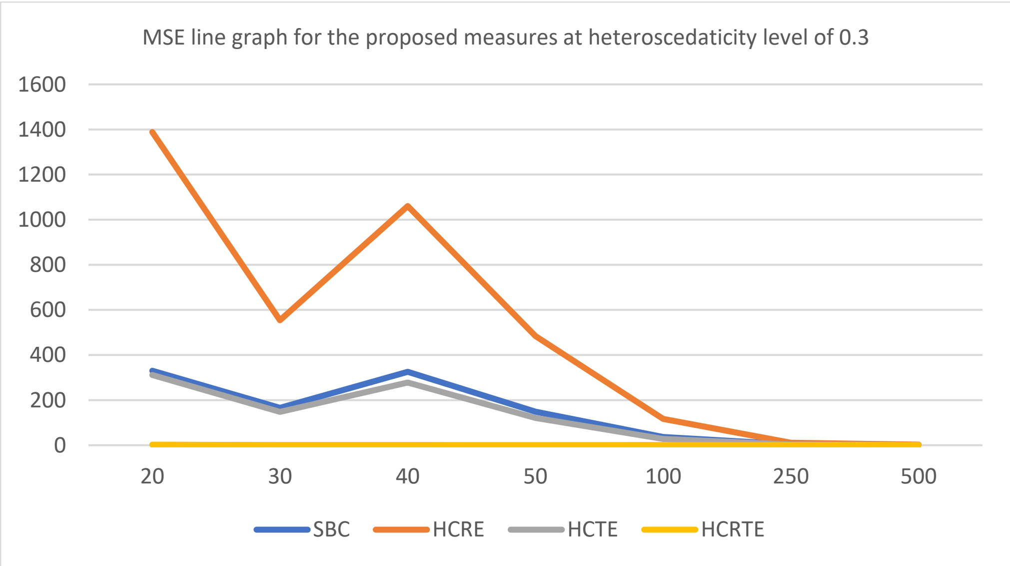
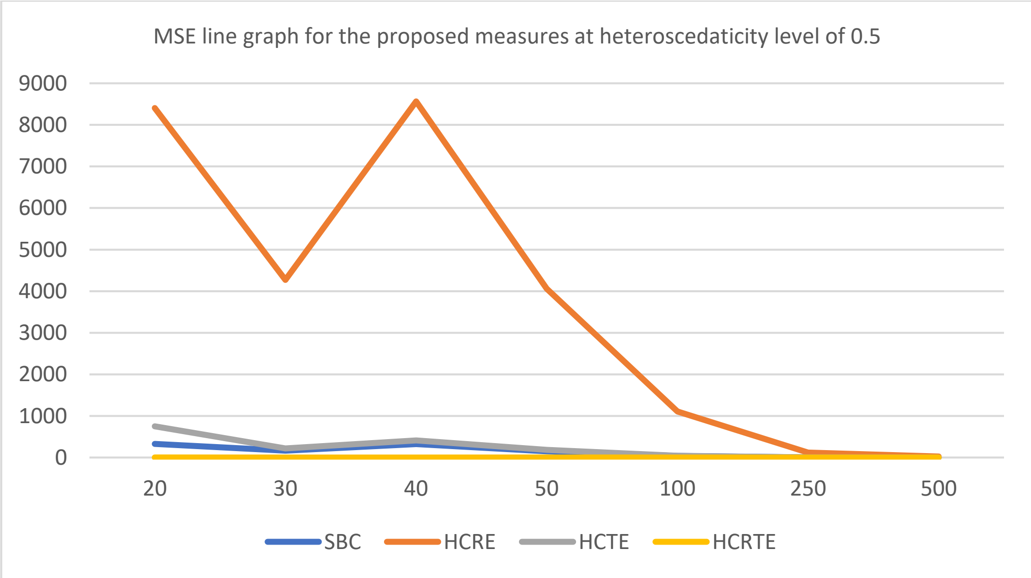


Figure 4.4 MSE Line Graph for the Proposed Measures at the heteroscedasticity level of 0.5



**The measure with the line closest to zero is the best*

From Table 4.2 and Figure 4.2. 4.3, 4.4, the following observations were made: Among the SFA correction measures (SBC, HCRE, HCTE, and HCRTE), HCRTE consistently exhibited the minimum MSE across all sample sizes (20, 30, 40, 50, 100, 250, and 500) and for all levels of heteroskedasticity (0.1, 0.3, and 0.5). This strongly suggests that the proposed correction measure (HCRTE), which addresses heteroskedasticity in both SFA error components, consistently yielded the lowest MSE values across all sample sizes, making it the most effective correction measure.

Conclusions

In conclusion, the effectiveness of heteroscedasticity corrections in the ordinary least squares (OLS) and stochastic frontier analysis (SFA) models is more pronounced in smaller sample sizes ($n < 100$), as evidenced by the simulation study results that indicate improvements in parameter estimates and reduced mean squared error (MSE) values as sample sizes increase towards infinity. Among the OLS models (OBC, OCRE, OCTE, OCEP), the OCEP model demonstrates superior performance with consistently minimal MSE across varying sample sizes. Similarly, in the SFA models (SBC, HCRE, HCTE, and HCRTE), the HCRTE model stands out with the lowest MSE values across different sample sizes.
However, corrections for heteroskedasticity in known forms (OCRE, OCTE for OLS; HCRE, HCTE for SFA) do not always yield better results compared to the jointly corrected OCEP and HCRTE models. This suggests that jointly correcting heteroscedasticity in both the u and v disturbance components of the model is more effective, especially in real-life data scenarios where the true form of heteroscedasticity may not be known. The results consistently show that OCEP and HCRTE provide the most efficient estimation, as indicated by their minimal MSE values at various levels of heteroskedasticity.

Based on these findings, it is recommended to test for heteroskedasticity in production models before applying any correction procedure and estimating SFA models. Furthermore, the use of relatively small samples (n < 100) can lead to more consistent and reliable results in estimating technical efficiency and optimizing the performance of the production model. The proposed estimators OCEP and HCRTE show promising performance in the simulation study, highlighting their potential for practical use in addressing heteroskedasticity and improving technical efficiency measures in SFA.

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