THE NET PRESENT VALUE AND THE OPTIMAL SOLUTION OF LINEAR PROGRAMMING IN INVESTMENT DECISIONS

VESA Lidia
Doctoral School of Economic Sciences, Faculty of Economic Sciences, University of Oradea, Romania
lidiavesa@gmail.com

Abstract: *This paper presents a comparison between the net present value (NPV) and the optimal solution of the linear program in order to offer an alternative perspective on the decision process. In the decision process, companies have to use more tools in order to make the right decision and to increase their values. So, using these two tools, namely, the net present value and the solution of the optimization problems, the companies will put together the expected benefits of the fixed asset investments and the available or potential resources. Using only one of these tools means that the company is oriented either to the future benefits of the fixed asset or to the investment capacity, with all technical or financial restrictions. The NPV is determined by using the standard formula while the optimal solution for the resource allocation is obtained by using the Simplex Algorithm and The Big M Penalty method. The comparison and combination of these indicators are used in the company's acquisition process and create some debates on the results in the acquisition process. The significant advantage of this paper is the improvement of the decision process in acquisitions by providing information from both the internal business environment and the external environment. Also, this comparison combines technical and financial information, which will make the decision of acquisition more reliable. There are some limits to this research. One limit is that it does not consider the possibility of delaying the investments since the NPV compares the now-investing to never-investing attitudes. Another limit is that the Simplex Algorithm offers a restrictive horizon of the decision since its components are expressed in positive integers. These two disadvantages may be discussed in further research, firstly, by appealing for the cost delay for making the right decision at the right time, and secondly, by using the fuzzy number in order to make the decisions in the fixed assets acquisition process more flexible. This last recommendation could replace the sensitivity analysis, which is a more complicated way to make the decision more flexible.*

Keywords: *net present value (NVP); investment; fixed assets; Simplex Algorithm; decision process; optimal solution.*

JEL Classification: *G17; G31.*

1. Introduction

Companies are often involved in complex decisions that ask for the management of information in such a way to achieve the best combination of limited resources and unlimited opportunities or needs. The need for fixed assets with increased

performance should be balanced with future possible economic benefits and with the available resources, or the cost of the resources in order to obtain the assets. Of course, more factors influence the decision: the production efficiency, the cost of the training, the payback period, the expected return.
The purpose of this paper is to offer an improvement of the fixed asset acquisition decision by comparing the net present value with the optimal solution of the linear program, solved with the Simplex Algorithm. The major contribution is the combination of these tools, which will help the companies to consider both future benefits and current limited resources, not only the financial resources but also the employee training and development, the available space, etc.

2. Short literature review

Several articles and books talked about the combination of the NPV and the linear optimization problems, but they proposed different ways than what this paper is going to introduce here. Table 1 summarizes the publications that put together these two different tools.

Table 1: The comparison between NPV and the optimal solution in literature review

Name of the authors	Method of using NPV and optimal solution
Okoye (1998)	They used the Simplex Algorithm to maximize the net present value subject to the budget constraints for each period of the present value.
Padberg and Wilczak (1999)	They used the Simplex Algorithm to maximize the net present value of the projects and proposed a new model by formulating the objective function as the difference between the horizon value of the project (or net present value) and the amount of money borrowed at the beginning of the period. The objective function is a cumulative function of the differences determined for each project.
Schwindt (2005)	He proposed using the Simplex Algorithm for maximization the NPV, taking into account a limited initial budget, and used it in building industry where the benefits gained from completed projects serve to finance the upcoming projects.
Watson and Head (2007)	They proposed using the Simplex Algorithm for choosing the investment project, if it is expected that the investment funds can be restricted in more than one period. Using the mathematical model for linear programs, the company should easily identify the project that satisfies the restrictions.
Aman (2019)	He used the Simplex Algorithm in order to maximize the NPV, offered interpretation on the slack variables and used the sensitivity analysis.

3. Investment decision using the Net Present Value

3.1. Definition and formula

The net present value is defined as ”the sum of the present values of the incoming (benefits) and outgoing (costs) cash flows over a period of time. NPV can be described as the difference between the sums of discounted cash inflows and cash outflows.” (Gaspars-Wieloch, 2017)

This financial tool is determined by the formula designed by Brealey, Myres, and Allen (2011):

$$NPV = C_0 + \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} ,$$

where: C_0 - the cash flow at time 0 (negative number);
C_t - the cash flow at the time t;
r – discount rate.

There is a rule of rejecting or accepting the investment projects from the NPV perspective:

1. If the NPV>0, the company should invest in the project;
2. If the NPV<0, the company should not invest in the project;

If there are two or more projects, the net present value of the combined investment is according to Brealey, Myres, and Allen, (2011):

$$NPV(A_1 + A_2 + ... + A_n) = NPV(A_1) + NPV(A_2) + ... + NPV(A_n)$$

- If the net present value is positive for all projects, the adding-up property is valid;
- If the net present value is positive for some projects and negative for others, this property may be tricky, because the companies do not know if the package of the investments with positive and negative NPV will be more favorable than investing only in the projects with positive NPV.

3.2. Example

Let us consider company X, which must purchase new equipment for the production process in order to replace the old one and to increase the performance and the efficiency of the current activity. The four types of equipment considered here are analyzed from the NPV perspective. The following table presents the data for these four fixed assets:

Table 2: The NPV for each equipment

Eq$_i$	C_0	C_1	C_2	C_3	C_4	C_5	**P.P.**	**NPV at 10%**
A_1	-10.000	3.000	4.500	3.500	800	0	2	-379
A_2	-15.000	5.000	7.000	9.000	8.000	7.500	3	12.221
A_3	-20.000	5.000	7.000	10.000	13.000	15.000	3	16.035
A_4	-27.000	7.000	8.500	11.000	13.000	16.000	4	13.464

where: Eq – equipment of type *i*;

C_t - the cash flow at time t, $t = \overline{1,5}$;

P.P. – payback period (it has an important contribution in investments, because it is compared with the cutoff period.)

From Table 2, it can be noticed that the equipment A_2, A_3, A_4 should be purchased now since the NPV is positive, and the aquisition of equipment A_1 should be rejected, due to the negative value of the NPV.

4. Investment decision using the Simplex Algorithm

4.1. Definition and methodology

The Simplex Algorithm is defined as "a step by step arithmetic method of solving linear programming problems, whereby one moves progressively from say a position of zero production and therefore zero contribution until no further contribution can be made. Each step produces a feasible solution and each step is an answer better than one before it, either greater contribution in maximizing problems or smaller costs in minimizing problems" (Okoye,1998).

In order to use the Simplex Algorithm, it is necessary to formulate the linear program as a mathematical model. The construction of the linear program is described in following table.

Table 3: The Elements of Linear Program

Components	Mathematical Model	Explanations
1. Objective function	$f(x) = c_1x_1 + c_2x_2 + c_3x_3 + \ldots + c_nx_n$	c_n – the coefficients of the objective function x_i, i $=\overline{1,n}$ - the variables of the problem
2. Restrictions	$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le b_2 \\ a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n}x_n \le b_3 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le b_m \end{cases}$	a_{ij}, i $=\overline{1,n}$ - the coefficients of restrictions b_i, i $=\overline{1,m}$ - right hand side value of the constraint
3. Nonnegativity conditions	$x_1 \ge 0$, $x_2 \ge 0$, ... , $x_n \ge 0$	

After the formulation of the linear program, there are some steps that should be followed in order to obtain an optimal solution for the linear program.

Table 4: The steps of Simplex Algorithm adapted from Bolos et al. (2020)

Steps	Mathematical description
1. Converting the objective function and restrictions - Adding slack variables - Adding artificial variables (Big M Penalty)	$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + s_1 = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + s_2 = b_2 \\ a_{31}x_1 + a_{32}x_2 + \ldots + a_{3n}x_n + s_3 = b_3 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + s_n + A_n = b_m \end{cases}$
2. Entering the restrictions in the Simplex Table	
3. Selection of the entering variable (optimality condition)	Minimizing problem: - If all differences: Z_j - $C_j \le 0$ – the program is optimal; - If there is at least one Z_j - $C_j > 0$ – the entering variable is x_i = max (C_j - Z_j) where: C_j - objective function coefficient, when j = $\overline{1,n}$; $Z_j – \sum C_b \times P_k$ where: C_b – the coefficient that each variable that appears at base has in the objective function; P_k – the coefficients of the variables in the restrictions when k= $\overline{1,n}$ (Table 5)
4. Selection of leaving variable	Minimizing problem: - the leaving variable is: $x_l = \min \left\{ \frac{P_0}{P_h} \right\}$ where: P_0 – the right-hand side value of the constraints (the table notation) P_h – the coefficients of the restrictions for the entering variable k= $\overline{1,n}$ (Table 5)
5. Updating the table and solving the iterations, until the program is optimal.	

Table 5: Simplex Table adapted from http://www.phpsimplex.com/simplex/simplex.htm?l=en

B_c	C_b	P_0	c_1	c_2	c_m	c_k	c_β	c_n
			P_1	P_2	P_m	P_k	P_β	P_n
P_1	c_1	B_1	1	0…	0…	a_{1k}	$a_{1\beta}$	a_{1n}
P_2	c_2	B_2	0	1…	0…	a_{2k}	a_{2v}	a_{2n}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
P_α	c_α	B_α	0	0	0	$a_{\alpha k}$	$a_{\alpha\beta}$	$a_{\alpha n}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
P_m	c_m	B_m	0	0	1	a_{mk}	$a_{m\beta}$	a_{mn}
Z_k		Z_0	Z_1	Z_2	Z_m	Z_k	Z_β	Z_n
$\Delta_k = Z_k - C_k$		Δ_0	Δ_1	Δ_2	Δ_m	Δ_k	Δ_β	Δ_n

4.2. Example

Let us consider the same company X from the previous example, with the same need to purchase four equipment for the production process. They are analyzed by a series of acquisition criteria, and by restrictions resulting from the company`s activity.

To obtain the optimal solution that will show the right answer in the decision process, consider how the following table presents the formulation of the acquisition problem.

Table 5: The Coficient`s Values of the objective funtion and constraints

Elements	Criteria	Notations	A_1	A_2	A_3	A_4
Objective function	Number of the assets	x_i	x_1	x_2	x_3	x_4
	Acquisiton Cost (min)	$Ca(A_i)$	€10.000	€15.000	€20.000	€27.000
Restriction 1	The surface/equipment	$s(A_i)$	265m²	240m²	300m²	275m²
	The Total Surface	S			2000 m²	
Restriction 2	Acquisiton Cost (min)	$Ca(A_i)$	€10.000	€15.000	€20.000	€27.000
	The Total Budget	B_c			€ 70.000	
Restriction 3	Trainig cost/equipment	$t(A_i)$	€2.000	€1.000	€2.800	€2.200
	Total Trainig Budget	B_t			€ 10.000	
Restriction 4	The obsolete assets that should be replaced				3 Eq	

The mathematical model for the linear program is as follows:

Objective function:

$$f(x) = 10.000x_1 + 15.000x_2 + 20.000x_3 + 27.000x_4$$

Constraints:

$$\begin{cases} 265x_1 + 240x_2 + 300x_3 + 275x_4 \leq 2.000 \\ 10.000x_1 + 15.000x_2 + 20.000x_3 + 27.000x_4 \leq 70.000 \\ 2.000x_1 + 1.000x_2 + 2.800x_3 + 2.200x_4 \leq 10.00 \\ x_1 + x_2 + x_3 + x_4 \geq 3 \end{cases}$$

Non-negativity conditions

$x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$

Once the mathematical model is adopted, the first step of the Simplex Algorithm is converting the objective function and the restrictions.

$$f(x) = 10.000x_1 + 15.000x_2 + 20.000x_3 + 27.000x_4 + 0S_1 + 0S_2 + 0S_3 + 0S_4 - MA_1$$

$$\begin{cases} 265x_1 + 240x_2 + 300x_3 + 275x_4 + S_1 = 2.000 \\ 10.000x_1 + 15.000x_2 + 20.000x_3 + 27.000x_4 + S_2 = 70.000 \\ 2.000x_1 + 1.000x_2 + 2.800x_3 + 2.200x_4 + S_3 = 10.00 \\ x_1 + x_2 + x_3 + x_4 - S_4 + A_1 = 3 \end{cases}$$

where: - S_1, S_2, S_3, S_4 – the slack variables
- A_1 – artificial variable

The slack variables meaning is:
- S_1 - the unused surface in m^2;
- S_2 – the unallocated budget;
- S_3 – the reduction of the training cost.

In addition to the slack variables, it was also introduced the artificial variable, because the problem is not a standard one. Actually, it is a non-standard with both ≤ and ≥ inequalities. So, it was necessary to add the artificial variable, which had no physical meaning and was introduced only for obtaining a basic feasible solution. To avoid having an artificial variable in the optimal solution, a very large penalty *M* was introduced in the objective function, as a positive constant value. (Dantzig and Thapa, 2002).

Moving on, the problem will involve two iteration: the initial Simplex Tableau and the second Simplex Tableau, in order to obtain this optimal solution:

$x_1=3$, $x_2=x_3=x_4=0$

This optimal solution means that company should purchase the asset A_1 because it satisfies the restrictions and minimizes the objective function, so $f(x) = 30.000$.

The slack variables are: S_1 = 1.205 m^2, S_2=€40.000, S_3=€4.000. This means that the company should re-allocate the funds for investment and calculate the cost of unused surface if the optimal solution is accepted, in order to evaluate the opportunity cost.

There are some advantages and disadvantages of using the Simplex Algorithm. The most important advantage is the opportunity cost that the algorithm reveals: the solution minimizes the objective function, but it is the surface that remains unused and the budget that should be allocated again. The company can estimate the cost of these slack variables and compare them with the optimal solution.

5. Discussion

There are three cases:

A. NPV > 0, optimal solution = 0. This is the case for the assets A_2, A_3, A_4, where: NPV(A_2) = 12.221, NPV(A_3) = 16.035, NPV(A_4) = 13.464 and the optimal solution is 0. The NPV is a tool that influences the rejecting or accepting of the acquisiton project: if the NPV is negative, the company should reject the project, and if the NPV is positive, the company should accept it. From the NPV perspective, the company should accept these three assets, but from linear programming solution, the company should reject them, because the optimal value is equal to 0.
B. NPV < 0, optimal solution > 0. This is the case for the asset A_1, where, NPV(A_1)=-379 and the optimal solution is 3. The company should reject the acquisition of asset A_1 from the NPV perspective and accept it from the Simplex solution. In this case, the company has to choose between the NPV solution and the optimal solution. The total NPV for the optimal solution is equal to: NPV(3A_1)=-1137.
    - If the costs of the slack variables (unused budget, unused surface) are smaller than the future economic benefits of the asset, the company should not invest in them.
    - If the costs of the slack variables are greater than the future economic benefits of the asset, the company should not invest in them.
C. NPV > 0, optimal solution > 0. This case is the ideal one and means that from both NPV and optimal solution perspective, the company should accept the acquisition of the asset because it satisfies the objectives and the constraints of the company and will generate the increase of the company`s value.

In order to solve the first case and to combine the solutions from the NPV and the Simplex Algorithm, the company can appeal to the sensitivity analysis, which is defined as the “way to determine how robust proposed solutions are to changes” (Dantzig and Thapa, 2003). This analysis has been used until now for three reasons:

- Testing the reliability of the results in case of significant changes in objective function and constraints;
- To present the relationship between the input and output components;
- To make the results more credible for the companies and to make the decision process more flexible. (Dhand and Singla, 2016).

In this paper, the sensitivity analysis has a new approach: the company uses this tool to evaluate the impacts of accepting the assets with positive NPV and 0 optimal solution. The sensitivity analysis will help the company to establish what the surface, the budget and the training cost should be, to accept the assets with 0 optimal solution, and positive NPV. But if the company do not intend to modify the constraints, because the available surface is the maximum surface available for the equipment, the allocated budget is the maximum budget that can be allocated, and the total training cost is the maximum cost that the company can afford. The easiest way to simulate it is to modify the fourth restriction, which is the replacement of the obsolete assets.

5.1. Example with NPV > 0 and optimal solution > 0

If the same company X considers that purchasing of the Asset A_3 and A_4 is the best decision because they have the highest total NPV: NPV (A_3+A_4)=29.499, the company can modify the fourth restriction and obtain the following linear program:

$$\begin{cases} 265x_1 + 240x_2 + 300x_3 + 275x_4 \leq 2.000 \\ 10.000x_1 + 15.000x_2 + 20.000x_3 + 27.000x_4 \leq 70.000 \\ 2.000x_1 + 1.000x_2 + 2.800x_3 + 2.200x_4 \leq 10.00 \\ x_3 + x_4 \geq 3 \end{cases}$$

It can be observed that the company limited the possibility to obtain an optimal solution that has the negative NPV. So, the solution with $x_1>0$ was almost eliminated. Solving the new linear program with the Simplex Algorithm, the solution is:

$x_1=x_2=x_4= 0$, $x_3= 3$

The company should acquire A_3 to get the objective function $f(x)=60.000$. The cost of acquisition will be greater than the cost from the first linear program, but the slack variables, the unused/unconsumed resources have a smaller opportunity cost: S_1 = 1.100 m^2, S_2=€10.000, S_3=€1.600. Even if the cost is greater, this new linear program proposes a better allocation for the resources.

5.2. Example with increased cost of unused resources

This sensitivity analysis is an uneasy and uncomfortable tool because it needs to solve the linear problem with the Simplex Algorithm every time when the constraints or the objective function change. If the company is in favor of acquiring other assets, the company should modify again the fourth restriction with percentage coefficients and reiterate with the Simplex Algorithm until the optimal solution is obtained.

$$\begin{cases} 265x_1 + 240x_2 + 300x_3 + 275x_4 \leq 2.000 \\ 10.000x_1 + 15.000x_2 + 20.000x_3 + 27.000x_4 \leq 70.000 \\ 2.000x_1 + 1.000x_2 + 2.800x_3 + 2.200x_4 \leq 10.00 \\ x_3 + 2x_4 \geq 3 \end{cases}$$

It can be noticed that the company was in favor of the A_4 and modified the fourth restriction. After solving the problem, the optimal solution is:

$x_1=x_2=x_3= 0$, $x_4= 1,5$

The slack variables are: S_1 = 1587,50 m^2, S_2= €29.500, S_3= €6.700. This example has the cost of unused resources greater than the previous simulation. The company should evaluate the optimal solution if it is a favorable choice because the unused resources would probably generate expenditures that will exceed the future benefits.

6. Conclusions and recommendations

The comparison and combination of the NPV and the Simplex Algorithm solutions with the help of the Sensitivity Analysis have some advantages:

- Increase the flexibility of the decision process and consider the preferences of the managers in this process;
- Increase the validity of the decision process, due to the different tools used to obtain the best solution;
- Put together the future and the present, by using the NPV to determine the future value of the assets and by using linear programming to obtain a solution that considers not only financial but also technical and operational constraints;
- Consider the internal and external business environment, by relating the NPV to the market and by using internal restrictions in the linear program to obtain the solution.

Though it has these advantages, the combination process is an uncomfortable and slow process, due to the sensitivity analysis that implies new iterations whenever the constraints change. It is the principal drawback of this paper, but it could be removed by using the Dual Simplex Algorithm and Fuzzy Linear Programs. The Dual Simplex is very used in re-optimization because it is a shorter way than the Primal Simplex Algorithm, so the company should not perform all iterations to obtain the optimal solution. The fuzzy linear programs allow a more flexible decision process because their variables, constraints, and objective function use the interval-valued fuzzy numbers (Guo, S. & Song, T., 2009). This is an advantage for solving a problem decision in an uncertain context when the constraints are changing in a short time, and it is necessary to use a “shortcut.”

Further research is needed to investigate the impacts of the Dual Simplex Algorithm or the Fuzzy Simplex Algorithm and the NPV combination on the decision process. Also, it would be a new avenue or research to create a combination between the Fuzzy Dual Simplex Algorithm and the NPV.

References

1. Boloș, M.-I., Bradea, I.-A., & Delcea, C. (2020). Linear Programming and Fuzzy Optimization to Substantiate Investment Decisions in Tangible Assets. *Entropy*, *22*(1), 121. doi: 10.3390/e22010121, [online], Available: https://www.researchgate.net/publication/338692423_Linear_Programming_and_Fuzzy_Optimization_to_Substantiate_Investment_Decisions_in_Tangible_Assets/citation/download

2. Brealey, R. A., Myres S. C., Allen F. (2010). Principles of Corporate Finance, (10th edition), New York City, NY: McGraw-Hill, [online], *Available: http://www.competitiontribunal.gov.au/__data/assets/pdf_file/0004/28246/END.042.001.0013.pdf*

3. Dantzig, G. B., Thapa, M. N. (2003). Linear Programming - Theory and Extensions. New York: Springer-Verlag, ISBN: 978-0-387-21569-3
4. Dhand, S. & Singla, A.. (2016). Sensitivity Analysis and Optimal Production Scheduling as a Dual Phase Simplex Model. *Indian Journal of Science and Technology*. 9(39). DOI: 10.17485/ijst/2016/v9i39/100788, [online], Available: https://www.researchgate.net/publication/299509470_Sensitivity_Analysis_and_Optimal_Production_Scheduling_as_a_Dual_Phase_Simplex_Model
5. Gaspars-Wieloch, H. (2017). Project Net Present Value estimation under uncertainty. *Central European Journal of Operations Research*, *27*(1), 179–197. DOI: 10.1007/s10100-017-0500-0, [online], Available: https://link.springer.com/article/10.1007/s10100-017-0500-0#citeas
6. Khan, A. (2019). Capital Budgeting and Improvement Process. *Fundamentals of Public Budgeting and Finance*, 275–318. DOI: 10.1007/978-3-030-19226-6_8, [online], Available: https://books.google.ro/books?id=Ie69DwAAQBAJ&printsec=frontcover&dq=fundamentals+of+public+budgeting+and+finance&hl=ro&sa=X&ved=0ahUKEwj-qrKzzoXpAhUrmYsKHR53ByEQ6AEIKDAA#v=onepage&q=NPV&f=false
7. Okoye, E. (1998). Application of Simplex Method to Accounting Decision Making. *Journal of the Management Sciences*. 1(3). 168-182., [online], Available: https://www.researchgate.net/publication/320627401_APPLICATION_OF_SIMPLEX_METHOD_TO_ACCOUNTING_DECISION_MAKING
8. Padberg, M., & Wilczak, M. (1999). Optimal project selection when borrowing and lending rates differ. *Mathematical and Computer Modelling*, *29*(3), 63–78. DOI: 10.1016/s0895-7177(99)00030-8, [online], Available: https://www.researchgate.net/publication/257296944_Optimal_project_selection_when_borrowing_and_lending_rates_differ
9. Schwindt, C. (2005). Resource Allocation in Project Management. Springer, DOI: 10.1007/3-540-27852-4
10. Sizong, G., & Tao, S. (2009). Interval-Valued Fuzzy Number and Its Expression Based on Structured Element. *Advances in Intelligent and Soft Computing Fuzzy Information and Engineering Volume 2*, 1417–1425. DOI: 10.1007/978-3-642-03664-4_150, [online], Available: https://www.researchgate.net/publication/220970203_Interval-Valued_Fuzzy_Number_and_Its_Expression_Based_on_Structured_Element/citation/download
11. Watson, D., Head, A. (2007), Corporate Finance. Principles and Practice, (4th edition), Pearson Education Limited, [online], Available: https://books.google.ro/books?id=qyd8yKFWjHUC&pg=PA173&lpg=PA173&dq=simplex+algorithm+compared+with+NPV&source=bl&ots=zvnEFRleuL&sig=ACfU3U10JQSyDJjARBd-2VO6vqCM6qBd4w&hl=ro&sa=X&ved=2ahUKEwiYzM6Fn4PpAhXvs4sKHfcUBuQQ6AEwBHoECAgQAQ#v=onepage&q=simplex%20method%20and%20NPV&f=false