KNOWLEDGE COMPLEXITY REACTION-DIFFUSION EQUATIONS WITH APPLICATION TO EASTERN EUROPE

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Abstract: *Nowadays, there is an increasing focus on knowledge input and output measurement rather than assessing the characteristics of the knowledge itself. This phenomenon happens since knowledge becomes central into capitalistic production processes: the competitive advantage of countries is the principal engine for the production of high-value, non-ubiquitous and complex knowledge (see Dicken, 2007). In this paper, we present the reaction-diffusion equations and their application to a particular economic phenomenon: in which way knowledge complexity diffusion might affect or not affect the labour productivity across the Eastern European countries. More profoundly, the aim followed by the present research work is to provide Eastern European countries with a substitute approach: a strategy. This strategy has the objective of enabling these countries to avoid acting as knowledge predators towards other countries (i.e. those such as Sweden, Finland and Denmark, referred to as prey countries, which invest their resources in knowledge and therefore in technology). We simulate the knowledge competition and the spill-over productivity effects across the Eastern and some European countries using specific reaction-diffusion equations types: the Lotka-Volterra equations which are able to describe a complex dynamic system of the prey-predator phenomenon. Our approach is based on the two steps of analysis. In essence, the first step, we compute the Knowledge Complexity Index of patents, for quantifying the European knowledge complexity; describing the possible spatial patterns and transformation of the European knowledge. Furthermore, we use the canonical Lotka-Volterra models of patents from the European Patent Office and the labour productivity data from the Eurostat database from 2000 and 2017 for Eastern European countries and some European countries, such as Finland, Sweden, Austria and Germany. More specifically, we have identified the presence of lagged knowledge between the Eastern European countries and some Northern European, such as Finland, Sweden and Denmark. In conclusion, the implementation of specific knowledge diffusion policy strategies could be helpful for future applications. It may avoid intra-guild cannibalism phenomenon from being engaged in some Eastern European countries.*

Keywords: *Knowledge diffusion; complexity nonlinear systems; Eastern Europe; reaction-diffusion equations; Knowledge Complexity Index.*

JEL Classification: *A12; B41; C02; C60.*

1. Introduction

Nowadays, complexity economists analysis emergent phenomena affirming that non-equilibrium is the natural state of the economy since the economy is in a permanent fluid state caused by fundamental uncertainty and technological innovation. In this theoretical framework, knowledge and, consequently, its creation and diffusion can be considered one of the essential productivity factors that have become the pivot into the capitalist production (see Asheim, 2005): the competitive advantage of countries derives from the production of high-value, non-ambiguous and complex knowledge. This characteristic set up knowledge as spatially closed, difficult to create or to move outside its production space. More specifically, according to Goldwasser (1985), knowledge complexity can be defined as follows: "[..] *An amount of knowledge that can be gained from communication by a participant with a polynomially bounded resource and investigate how much knowledge must be communicated for providing a theorem [...]"* (Goldwasser, 1985: 294-295). In other words, according to Goldreich, knowledge complexity "[...] measure the computational advantage gained by interaction [...]" (Goldreich, 1999: p.1). Within this theoretical framework, which type of countries hold the most valuable knowledge? This is not an easy question to answer because, according to Pavitt (1982), one of the principal reasons why researchers are generally unfamiliar with knowledge composition derives from the inaccurate measurement of knowledge and technology: to study the differences in the nature of knowledge over space, only few research have encouraged the assessment of the technological coherence (see Jaffe,1982) and the measurement of the technological distance between firms (i.e. Teece,1994). The understanding of the rise of this type of interactions is one of the traditional and principal goals of *Econophysics*. This is because these types of interactions become notable in economics at the macro-level. In essence, the attitude of these types of systems suggests the existence of symmetric feedback between their micro-structural and macro-structural elements: the macroeconomic time-series tend to be characterised by a lag-lead structure. Consequently, there is a shift towards investigating the assessment of how these complex network interactions occur between heterogeneous economic entities (here, Eastern European countries) related to the technological flow. The principal purpose of the present paper is to analyse how knowledge complexity might affect labour productivity across Eastern European countries. In order to achieve this goal, we apply the reaction-diffusion equations, specifically the *Lotka-Volterra equations* (hereafter LV), to shape the complex interdependence exhibited by the microeconomic variables used in terms of the inter-temporal technological diffusions.

Based on these premises, the present work is structured as follows. Section 2 attempts to understand the mathematical properties of the reaction-diffusion systems representing the foundation of the linkage description. A particular subtype, the Lotka-Volterra interaction-type system, is then discussed. Next, in Section 3, we present an empirical application of the LV model to study the spill-over effects on the Eastern European country entities using patents, and consequently the Knowledge Complexity Index, and data on labour productivity per hour worked from 2000 to 2017. Finally, Section 4 supplies final remarks and discusses the remaining open questions and future challenges presented by this avenue of economic research.

2. From the reaction-diffusion equations to the Lotka-Volterra model: knowledge predator-prey interactions

As stated in the Introduction, the reaction-diffusion equations are mathematical models applied in a wide range of subjects: a common principal characteristic of these systems is the possibility of their being expressed by partial differential equations that are semi-linear and parabolic in nature. Mathematically, the standard form of a reaction-diffusion equation is given by

$$\frac{\partial u}{\partial t} = D\nabla^2 u + R(u) \tag{1}$$

where *u(x,t)* represents the vector function to be calculated, *D* is the diagonal matrix of the diffusion coefficient, and *R* is assimilated to the local reaction function. In particular, in the reaction-diffusion *two-component (or two-dimensional) equation systems*, *u(x,t)* and *v(x,t)* are assumed to be the density functions of the two populations, and can be written as follows:

$$\begin{aligned} \frac{\partial u}{\partial t} &= Du\frac{\partial^2 u}{\partial^2 t} + F(u,v) \\ \frac{\partial v}{\partial t} &= Dv\frac{\partial^2 v}{\partial^2 t} + G(v,v) \end{aligned} \tag{2}$$

where the coefficients (*Du*, *Dv*) are the diffusion constants and the additive terms (*F(u,v),G(v,v)*) are the reaction functions. Furthermore, in Sirohi (2015) work, the non-linear coupled partial equations that identify the predator-prey interaction, are the following:

$$\begin{aligned} \frac{\partial u}{\partial t} &= \nu(1-u) - \frac{\alpha u v}{u+v} + \nabla^2 u \equiv f(u,v) + \nabla^2 u \\ \frac{\partial v}{\partial t} &= \frac{\beta u v}{u+v} + \gamma v - \delta v^2 + d\nabla^2 v \equiv g(u,v) + \nabla^2 v \end{aligned} \tag{3}$$

where *u=U(x,y,t)* and *v=V(x,y,t)* are the population densities of prey and predator, respectively, at the generic point in time (*t*). What is important to stress from this model are, at least, the following three features:

- The non-linear dependence might lead to endogenous dynamics in the form of perpetual oscillation, which are useful for describing microeconomic variable fluctuations;
- The dynamic system might not reach a constant equilibrium;
- The existence and uniqueness of the Cauchy problems. According to Mamontov (2013), it is possible to determine the precise analytical solution of the Cauchy problem for this type of equation. The author has confirmed that the problem mentioned above is equal to the following problem

$$\frac{\partial u}{\partial t} = \nabla_X^T (dt) \nabla_X u + a(t)u + b(t, X) \tag{4}$$

under the following constraints:

$$\lim_{t \downarrow t_0} \overline{u}_0$$

then, following the author's assumptions, the fundamental solution is as follows:

$$\varphi(t, y, s, z) = \rho(\sigma(t, S); y, z) \exp[\int_S^t a(w) dw] \tag{5}$$

where *a(w)* represents a real scalar function, and *w* is the correspondent real scalar variable. To conclude, this analytical description is possible to solve the Cauchy problem efficiently and demonstrate how the different terms of the model participate in it.

Furthermore, Ganguly et al. (2017) applied the Sirohi (2015) mathematical formulation in order to model the inter-dependency of the complex microeconomic variables between countries in terms of inter-temporal knowledge diffusion. In their work, they assumed as background the existence of the international technological frontier in which some countries emerge as predators and while others emerge as prey: prey countries invest their resources to develop newer knowledge, and consequently, technology, while the predator countries imitate this knowledge without engaging in original or newer innovation. Regarding this aspect, and in line with the authors, we have adopted this model because it has at least two principal advantages: first, it postulates an endogenous source of external non-linearity requirements; secondly, the model sets a time-lag between the predator and prey populations.

Subsequently, it is necessary to set out the following model hypotheses:

- *N* distinct economies are taken into consideration, where the leaders and the followers are referred to as *L* and *F* respectively, and economies are endowed with an equal potential to innovate;
- Within the *N* economies, there is a presence of households, and the output is produced using a combination of labour and capital;
- These countries' economies are *single-good* economies, and the fundamental scope of each entity in the system is to maximise his/her/their objective (profit/utility) function.

For the purpose of the present work, the LV equation for an *N-country scenario* is as follows:

$$\frac{\partial z^i}{\partial t} = \mu_{ij} z^i + \sum_{i \neq j} \mu_{ij} z^i z^j \tag{6}$$

where z^i is the *knowledge evolution term* that introduces the linkage across economies. In more depth, it represents the *knowledge shock process*, which is orthogonal to the microeconomic variables of each economy. Moreover, the evolution of knowledge is given by

$$Z(t+1) = T(z(t)) \tag{7}$$

where *T(z(t))* describes the interconnection terms. In more depth, to apply the model in the present empirical work, we use the *transformative function*. This function can be described as follows:

$$z^i = \frac{1}{1 + e^{-kS^j(t)}} \tag{8}$$

where *k* is the constant that controls the spread of the technology variables. The usage of this transformation reduces the volatility of the process (for a full description of the model see Chakraborti (2016)).

3. The knowledge complexity diffusion in the Eastern countries

The practical purpose of the research work is to determine whether if and in which ways the knowledge complexity diffusion might affect or not affect the labour productivity across Eastern European countries according to the mathematical

elaboration of Ganguly et al. (2017). As stated in the Introduction, one of the principal challenges for knowledge complexity measurement is the identification of technological opportunities. To achieve this goal, we have used patent data to evaluate the distances between classes of technologies. This usage provides at least two advantages: (i) it provides a visualisation of a local knowledge space; and (ii) it can examine how the space structure may lead the development of local knowledge trajectories across possible technological diversification structures.
Specifically, we used the European Patent Office database to identify the technological range of interest necessary to estimate the Knowledge Complexity Index (hereafter KCI) based on the *methods of reflection* developed by Hidalgo et al. (2009). Moreover, we also used the labour productivity per hour worked, as according to the Eurostat database description, this metric a better explanation of the productivity development in an economy, as it eliminates differences in the full time/part-time workforce composition across countries and years. In order to study this phenomenon, we have divided the analysis into two steps. In the first step, we have computed the KCI using *EconGeo*; an R statistical software package developed Balland (2016). According to him, this package proposes user-friendly functions able of computing a series of commonly used indices in Economic Geography and Economic Complexity. In the second step, we have computed the Lotka-Volterra equations for both the KCI and labour productivity variables using *prime*r, an R statistical software package.
Regarding the first step, as can be seen from Table 1, the KCI of the whole European countries is quite homogeneous over the considered period. Knowledge complexity is relatively high (KCI$>$70) (for example) Belgium, the Netherlands and Austria; these countries tend to develop several technologies that can only be replicated in a small number of other European countries. Knowledge production is also moderately high (60$<$KCI$<$80) in Greece, Hungary and Sweden. Knowledge production is considerable low (KCI<50) in Romania, Bulgaria, Czechia, Latvia, Lithuania and Slovakia.

Table 1: The Knowledge Complexity Index (years: 2000 and 2017)

	2000	**2017**		**2000**	**2017**		**2000**	**2017**
BEL	100.00	100.00	HRV	00.00	18.34	POL	3.49	22.80
BGR	100.00	4.14	ITA	96.39	0.00	PRT	34.66	53.64
CZE	00.00	00.00	CYP	62.62	100.00	ROU	00.00	00.00
DNK	86.14	00.00	LVA	21.97	45.10	SVN	32.89	100.00
DEU	100.00	100.00	LTU	0.00	00.00	SVK	24.50	39.38
EST	0.00	93.01	LUX	100.00	93.30	FIN	47.36	2.32
IRL	83.40	31.68	HUN	67.67	57.33	SWE	68.91	00.00
GRC	31.44	60.52	MLT	92.25	80.80	GBR	53.92	3.00
ESP	37.21	100.00	NDL	100.00	93.30			
FRA	100.00	89.02	AUT	78.43	81.92			

Source: Authors' computation

Focusing on the second step, we compute the LV equations in order to study the different micro-variables entailed within the system description, the former soviet group of countries. According to Chewing (2002), they have manifested a reasonable number of interactions and/or spill-over effects in a time-honoured fashion over the past years. Fig.1 presents the transformative function of KCI and the analogue of labour productivity per hour worked variables used during the period of analysis under consideration. In these graphs, the x-axis represents the time-point at which integration is achieved, while the y-axis symbolises the populations' vector at each time *t*; the lines of different colours represent different European countries. It is possible to stress out that in the interests of achieving the highest possible graphical clarity, we have opted to represent only those countries most significant to the scope of this paper's conclusions: Romania, Slovakia, Bulgaria, Hungary, Czechia, Latvia, the United Kingdom and Finland.

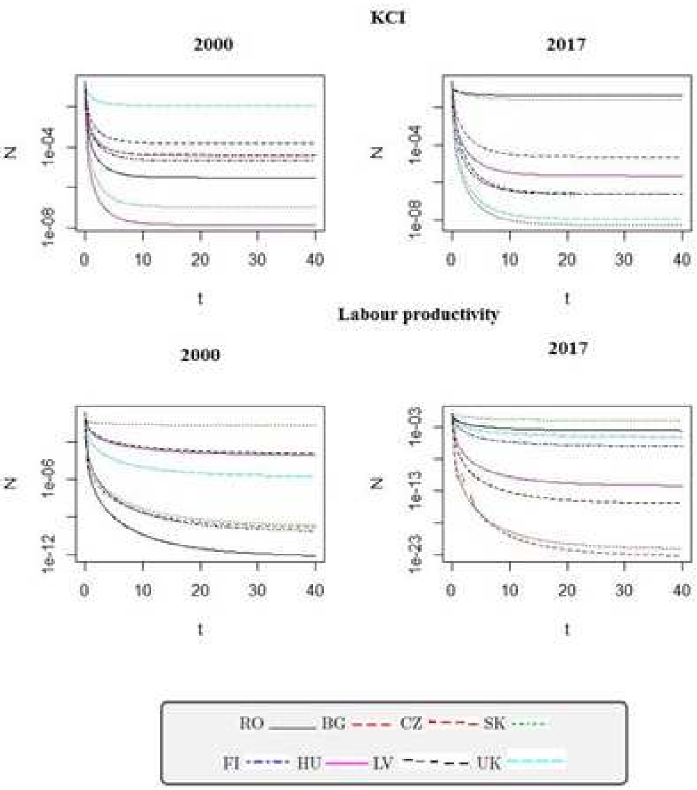

Figure 1: Evolution of the canonical LV model. The panel shows the KCI and labour productivity evolution under a functional transformation following Eq.8 with the turning parameter k=0.50

As can be seen from the graphs (Fig.1) above, during the period, the prey-predator interaction in terms of knowledge complexity which is characterised by a catching-up process undergone by the Eastern European low-income countries, such as Romania and Slovakia have not affected the labour-productivity prey-predator interaction process. In more depth, the KCI prey-predator interaction in 2017 exhibited some similarity with the labour productivity prey-predator interaction in 2000. Furthermore, some low-income European countries (i.e. Bulgaria) demonstrated *predator-like* conduct without notable enhancement behaviour throughout the considered period of analysis.

4. Conclusion

In the present research work, we presented a knowledge diffusion evolution paradigm using the Lotka-Volterra type of non-linear equations. The application of these equations establishes an alternative means of obtaining a cross-correlation analysis among the Eastern European countries in terms of knowledge complexity diffusion: lagged correlation structure between the leaders (such as The United Kingdom and Finland) and the followers (such as Romania and Bulgaria. In more depth, this has created a different shifting of the knowledge complexity into a more visible macro-variable: the labour productivity across Eastern Europe. If, on the one hand, this model can be handy for shedding light on the possible presence of lagged correlation structures, then, on the other hand, it is also possible to identify at least two significant problems with this simple model. Firstly, the present generalized framework may not guarantee convergence to a steady-state equilibrium. Secondly, the model is not possible to adequately describe the spill-over effects. This last aspect could well require more and future research development, particularly concerning its application to all European countries. One possible direction for this research might concern with uncovering the precise mechanisms through which the knowledge spill-over effects affect firm labour productivity. As firms can gather all necessary technology capabilities in a very successful way, it may be useful to involve more information exchange through the implementation of specific Eastern European projects, to avoid predatory and intra-guild cannibalism phenomena from being engaged in by some low-income countries.

References

1. Asheim, B. and Gertler, M. (2005) 'The geography of innovation: Regional innovation systems' in Faberberg, D., Mowery, David C. and Nelson, R. (eds.) *The Oxford handbook in innovation*, pp. 291-317, Oxford University Press.
2. Balland, P.A. (2016) "EconGeo: Computing Key Indicators of the Spatial Distribution of Economic Activities", *R package*.
3. Chakraborti, A. (2016) "Stochastic Lotka-Volterra equations: A model of lagged diffusion of technology in an interconnected world", Physica A: Statistical mechanics and its Applications Vol.442, No.15, pp. 214-223.

4. Chewing, Y.W. and Westermann, F. (2002) ‘Output dynamics of the G-7 countries-stochastic trends and cyclical movement’, *Applied Economics*, Vol.34, No.18, pp. 2239-2247.
5. Dicken, P. (2007) *Global shift: Mapping the changing contours of the world economy*, CA: Sage, Newbury Park, New York.
6. Eurostat database https://ec.europa.eu/eurostat/databrowser/view/TESEM160/default/table?lang=en [Available 18 April 2020].
7. Ganguly, S., Upasana, N., Chakraborti, A. and Chakraborti, A. (2017) ‘Reaction-Diffusion Equations with Applications to Economic Systems’ in Abergel, F., Aoyama, H., Chakrabarti, B.K., Chakraborti, A., Deo, N., Raina, D. and Vodenska, I. (eds.) *Econophysics and Sociophysics: Recent Progress and Future Directions*, Vol. 4128, pp. 131-144, Springer, Heidelberg.
8. Goldreich, S. and Petrank, E. (1999) “Quantifying knowledge complexity”, *Computational complexity*, Vol. 8, pp. 50–98.
9. Goldwasser, S., Micoli, S. and Rackoff, C. (1985) “The knowledge complexity of interactive proof-systems”, *STOC '85: Proceedings of the seventeenth annual ACM symposium on Theory of computing*, pp. 291-304.
10. Hidalgo, C. and Haussman, R. (2009) ‘The building blocks of economic complexity’ in *Proceedings of the National Academy of Science of the United States of America*, Vol. 106, pp. 10570-10575.
11. Jaffe, A.B. (1982) “Technological opportunity and spillovers of R&D: Evidence from firm's patents, profits and market value”, *American Economic Review*, Vol.76, No.5, pp. 984-1001.
12. Mamontov, E. (2013) “Exact analytical solution of the Cauchy problem for a linear reaction-diffusion equation with time-dependent coefficient and space-time dependent source term”, *Applied Mathematics Letters*, Vol.26, No.3, pp. 315-317.
13. Pavitt, K. (1982) “R&D, patenting and innovative activities: A statistical exploration”, *Research Policy,* Vol. 11, pp 33-51.
14. Sirohi, A.K., Banerjee, M., and Chakraborti, A. (2015) “Spatiotemporal pattern formation in a prey-predator model under environmental driving force”, Journal of Physics: Conference Series, Vol. 638.
15. Teece, D., Rumelt, R., Dosi, G. and Winter, S (1994) “Understanding corporate coherence: Theory and evidence”, Journal of Economic Behavior & Organization, Vol. 23, pp. 1-30.
16. Turing, A.M. (1952) “The chemical basis of morphogenesis”, *Philosophical Transactions of the Royal Society – Biological Sciences*, Vol.237, No. 641, pp. 37-72.