DEVELOPING OPTIMAL PRODUCTION STRUCTURE WITH LINEAR PROGRAMMING MODEL

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Abstract: *As nowadays, besides productions of food and forage materials, herbaceous and woody energy crops can be used to optimize the production structure, I have found it practical to present a model that is based on a Simplex algorithm. A multi-periodic linear programming model should be applied, if we would like to examine not only the traditional arable crops but also herbaceous and woody energy crops. In my article, the basic structure of the multi-periodic linear programming model and its theoretical background will be presented, with which, after filling it with corresponding data, an economic structure could be developed.*

Keywords: *linear programming; operations research; application; multi-periodic LP; cultivation.*

JEL Classification: *C60; Q19.*

1. Development and significance of operations research

Linear programming model is a part of operations research. It dates back to World War II in England, when a research team emerged to find solutions to military problems, such as carrying military supply substitutions. The name “operations research” originates from this, since the word “operation” refers to military operations (Bajalinov and Imreh 2001). Later, researches of this area realized that the results of previous researches could also be used in post-war peacetime. As an example, it became easy to find out the fastest ways to deliver products to customers and to develop locations of optimal military storage facilities. As a result of the progression of computers, operations research improved rapidly both in terms of content and scope. Operations research became a more and more recognized method in research departments of large corporations. Most of the operations research methods that are used nowadays have been developed and have become mathematically precise during these decades (Temese and Varró, 2007).
From that time, operations research has become a rather application-oriented science, which uses information to its methodological development from other sciences, such as mathematics, information technology, engineering and economics, so it is quite interdisciplinary. Therefore, operations research is one of the most significant and most versatile methods that is used to promote practical decisions in socio-economic life (Temesi and Varró, 2007).
It can be stated that operations research is an extremely comprehensive and complex science, so it is difficult to give a unified and complete definition of the method. Wayne L. Winston, author of *Operations Research: Application and Algorithms* says that operations research is a science that approaches decision making from a scientific point of view and it helps to determine the best structure and operation of a system, especially, when resources are scarce (Winston, 2003).

According to another well-known researcher, operations research is a discipline that deals with comprehensive preparation of optimal decisions and the method to determine the best way of realizing those decisions, primarily in economic, sociological, technological and military fields. Primarily, it uses mathematical models and it is closely related to electronic data processing (Ferenczi, 2006).
Operations research could be applied efficiently in many areas of business, such as production management, corporate strategy planning, financial management processes and corporate engineering developments (Temesi and Varró, 2007).
On most occasions, operations research is performed in an organization, for example in an institution, company, economic-technological unit etc. Within these organizations, everyday activities might cause conflicts that need to be solved (or improved, modified). In such cases, the organization needs to modify, improve, optimize its activity, which involves decision making. To make a decision, suggestions are needed, that is to say decision alternatives are needed. To process an alternative, the problem itself must be defined first.

1.1. Concept and significance of the model

The most effective method for preparing and making decisions is to apply different models. A model is the imaging of reality that represents the examined object, situation, process, usually with significant simplifications, focusing on the essence of the problem (Ábrahám, 2013). Therefore, the model is the simplified image of objective reality. Simplification grabs and highlights the essence. Thus, the model contains the most important components, properties and connections of the examined object (Ferenczi, 2006).
Consequently, in operations research, we create a model of a problem that emerges in real life, which represents the examined object materially and conceptually. So, the model allows to acquire new knowledge in the cognitive process and in the research; it could be an intermediate link between theory and reality (Csernyák et al., 1987).
There are various types of models for processing problems, as shown in Figure 1.

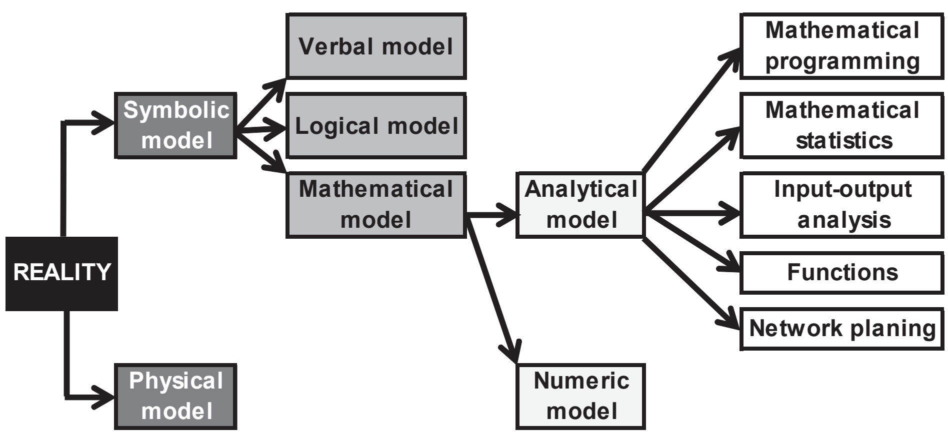

Figure 1: System of models according to their application
Source: Own-editing

As Figure 1 shows, models can be divided into two major groups. The first group contains physical models and material models. Their subgroups are the iconic and analogue models. An iconic model is the least abstract physical replica of reality; however, it cannot imitate its function and internal processes. Analogue models are closely associated with iconic models. The difference between them is that analogue models are able to imitate reality's internal contexts and its function. Tangibility and physical appearance are basic characteristics of these two models.
The second group consist of symbolic models that allows the conceptual imaging of reality. It has three subgroups: verbal models (use language tools to draw the problem, for example a debate), logical models (its most important elements are the logical symbols, could be used more efficiently to solve complex problems, have fewer errors than verbal models) and mathematical models (the best to solve high-level problems, as it is clear-out and thorough). Verbal models contain many errors and redundancies, so they are not adequate to solve complex socio-economic problems.
Mathematical models have further subgroups: numerical and analytic models. Numerical models consist of digits, they can be used to make certain forecasts, while analytic models consist of digits and logical symbols and they can give solutions to exact problems. Basically, this type of model is the best for imaging and solving complex problems (Nagy, 2014).
In my research, I examine mathematical programming (linear programming) within analytic models.

1.2. Development and practice of linear programming

The currently known scientific method, linear programming was developed to solve complex practical problems (Cabot and Erenguc, 1986). The solution of economic problems and tasks was the first major group of research areas, where the available resources were used in the most efficient ways to maximize the benefits. These problems were modelled with linear functions (Hirkó et al., 2000).
Linear programming task is a conditional extremity function, in which conditions are given in the forms of linear equation and inequation, and under these conditions we are looking for the minimum or the maximum of a linear function of the objective function (Bajalinov and Imreh, 2001).
Besides solving the problems that emerge in different areas of corporate management (delivery, product selection etc.), linear programming models can also be used expansively in the preparation of corporate plans (Mayer, 1998). The best-known subtype of linear programming models is the linear objective function optimization (Terlaky, 2001), which can be used in many different areas. For instance, in the case of manufacturing companies, determining the maximum revenue and appointing the profit-making product portfolio; in animal husbandry, it can be applied to make feed formulations that provide the most effective results; in road construction, to define the asphalt composition from which the best quality of asphalt could be made; creating food recipes with a given purpose (e.g. nutritional content); to solve portfolio problems (Hirkó et al., 2000).
Considering the above-mentioned issues, linear programming is a rather versatile scientific method, it is worthily one of the most essential applications of operations research. The method precisely expresses the principles of rational management: achieving the desired economic goal with the lowest possible input /input oriented/ and maximizing the income from the available resources /output oriented/). These

two terms are equivalent and match the methodology of linear programming. According to these, tasks of linear programming can be divided into two groups: “Maximum task” (principle of maximizing the income) and “Minimum task” (principle of minimizing the expenditure).
Tasks of linear programming could be done easily by using the Solver extension of Microsoft Excel software. Solver uses the simplex method, which is the most effective solving algorithm in linear programming, to perform optimization. The objective function and limiting conditions of optimization problems can be linear and non-linear. Solver solves linear equation systems with change of basis, while for non-linear problems, it uses the gradient method (Glevitzky, 2003). Solver adjusts the values in the decision variable cells to satisfy the limits on constraint cells and produce the result you want for the objective cell.

1.3. Linear programming model on optimizing the crop ratio

I have already mentioned in the previous section that operations research, in a narrow sense, is a scientific method that finds functional maxima and minima to prepare decisions and to define economic optimum (Csáki and Mészáros, 1981). Its features include linear and non-linear programming models, stockpiling models and network planning. Agriculture can be considered a specific field of linear programming, including the sector of crop production. The application of decision support systems is crucial in this area; its significant researchers are worth-mentioning: Dantzig, 1963; Csáki, 1969; Csáki and Varga, 1976; Szelényi, 1977; Dinya, 1978; Király et al., 1978; Vinczeffy, 1980; Csáki and Mészáros, 1981; Forgács, 1981; Tóth, 1978; Nemessályi, 1982; Ertsey and Tóth, 1985; Ertsey, 1974 and 1986b and Nagy, 2009. These researchers were engaged and were outstanding in the following activities: optimization of feed utilization, feed manufacturing, complex corporate planning, crop production technologies, automation of corporate planning.
In the beginning, the widespread use of linear programming in agriculture was limited by the capacity of computers and by the lack of knowledge of the method. After the 1968 economic reform, corporate independence and the introduction of the methods of operations research to agrarian higher education, new perspectives have opened for the practical application of these methods. In the 1970s and 1980s, the Debrecen operations research school, led by József Tóth, achieved significant results in processing planning procedures and their practical application supported by computer programs based on linear programming (Ertsey et al., 2002). In the early 90s, computers and information technology developed quite rapidly, which resulted in the progression of decision support. With the development of information technology, it became easier to establish models, which, through system-approached analysis, helped to establish economic decisions and helped to methodize the processes of agriculture.
It is important to note that operations research helps to prepare decision making, it supports the decision-making process; however, it is not its task to make the decision itself (Williams et al., 2005). Decision making is multiple-stage process, which has 6 different elements (Figure 2.).
During status analysis, the initial state of the corporate should be reviewed and the statistical analysis should be performed (analysis of variance, trend computation, regression analysis).

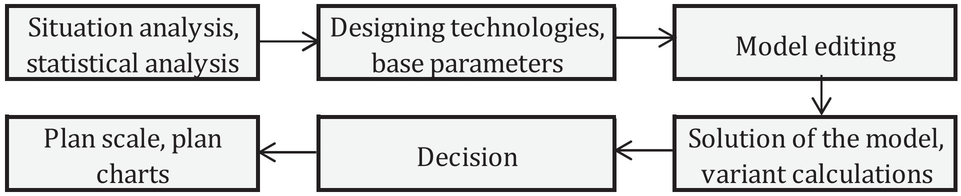

Figure 2: Multiple-stage process of decision preparation
Source: Own editing, based on ERTSEY et al., 2002

Conceptualizing includes the principles of production, utilization of resources, quality of production and market connections and limitations. In technology planning, a work plan, technological calculations, sector calculations have to be defined, just as the average yield of activities and the value of the objective function coefficient. By defining these, calculations could be automatized and the models could also be established.
The farming process, crop production, was built from sectoral systems; its general pattern is shown in Figure 3. The process of sectoral planning is based on databases, which are also known as master data. The two most important criteria in selecting master data, are the following: to have a decisive role in developing technological operation sequences and to attribute those sequences with features, which ensure the establishment of a connection between them so that the complete system could operate. The master database of land, power, machinery, manpower, work operation, plants and green crops should be developed under these circumstances. It is important to note, that these notions can only be considered as a whole system, they could not be examined separately.

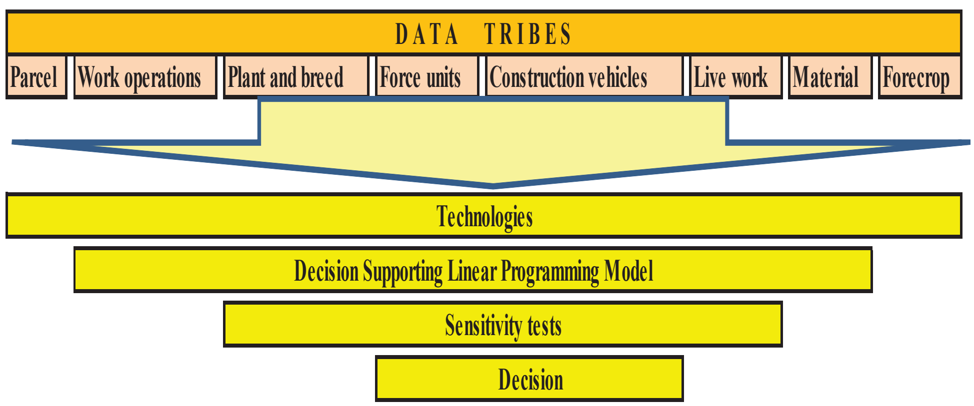

Figure 3: The process of decision preparation related to crop production planning
Source: Own editing, based on ERTSEY et al., 2002

Using master data, technological operations could be established that gives the time limit, which must be kept to ensure the optimal growth of the given plant. With these operations, we can make a plan for the given year and we can make the production values and expenses calculable (Ertsey et al., 2002; Ertsey and Fenyves, 2005)

The establishment of the production structure and the making of the related decision are the results of the process of decision-making and the planning process (Winston, 1997), which have the following well-defined and interconnected work phases:

- site quality, evaluation and analysis of the system of natural and economic resources;
- development of concepts;
- planning of technologies of the given sectors;
- establishing a mathematical model that describes the system of land use;
- developing different versions of decisions, developing alternatives and their analyses;
- the decision.

During the run of the base model, the least possible limit should be used. Developing the variants, only one parameter should be changed at a time, because in this way, influencing factors can be defined precisely. To frame versions of decisions, it is recommended to apply sensitivity analysis, since it could really help to make the final decisions.

1.3.1. Presentation of the objective function of linear programming models

In the model, it is recommended to make individual sectors compete based on their marginal contributions, as the individual crop production sectors contribute to the corporate income by the margin of their manufacturing output and the variable costs. By running the model, our aim is to establish a production structure that provides the highest income, or to achieve a given production structure using the lowest costs. If the model is optimized based on marginal contribution, which is the most common goal, then the coefficients of the model's objective function will show the extent of the contributions of activities to the corporate income.

Within variable costs, it is advisable to calculate material expenses (sowing-seed, cutting and pesticide), personnel costs, ancillary services and other direct costs (20% of variable costs).

Within ancillary services costs, fuel costs can be counted as a variable cost. According to the calculations of Pfau and Széles (2001), within the costs of maintenance and repair, 60-70% can be defined as material expenses and 30-40% as personnel costs. Depending on the form of the corporation and its operation, it is also necessary to take the public utility charges into account, which are usually not fixed**.** In model calculations, the percentage of the cost of ancillary services should be defined on the basis of the activities of the corporations. Therefore, within the cost of maintenance and repair, the percentage of variable costs is 85, while the percentage of fixed costs is 15.

1.3.2. Annual balance assumption of crop production LP model

Making a crop production model, it is always recommended to use a given size reference farm (x hectare), which have to be filled with parameters used in practice. In a crop production linear programming model, the following variables should be configured:

x_j: the planned acreage of the jth plant;

y_j: the jth resource;

β_i : the amount of the planned seasonal labor during the i^th period, usually in hours of shifts;

δ_b^h : the quantity of the h-typed machines to be purchased, in case of investment;

δ_i^h : the number of leased shift hours in the i^th period;

H_i : the amount of loan taken during the i^th period.

Values in the indexes could include the following values:

$$j = 1, 2, ..., n; \quad i = 1, 2, ..., m; \quad h = 1, 2, ..., u$$

Establishing a crop production model, the following balance assumption have to be established:

- Non-negativity, which is important because neither the solution nor the application can contain negative values. Balance assumption: $x_1, x_2, ..., x_j \geq 0,$ $y_1, y_2, ..., y_j \geq 0,$
- Balance assumptions related to land use are significant due to the proper utilization of the land. In the case of exquisite utilization of the land, the balance assumption is equal. Balance assumption: $\sum_{j=1}^{n} f_j x_j = F$, where f_j is the j^th specific area requirement of crop production value, while F means the available cropland. In some cases, an upper bound could be applied, for instance, when the land was not fully utilized, or if we have had different plans for some parts of the area. In this case, the balance assumption is $\sum_{j=1}^{n} f_j x_j \leq F$.

  In my model, the areal balance assumption is used to help to define how many hectares we would like to cultivate and whether we would like to include the effects of the preceding crop. Maize can be sowed in every 2 years to the same land, sunflower and autumn rapeseed in every 5 years and autumn wheat in every 3 years.
- Balance assumptions for labor are best considered in decades in the model, however, it is possible to examine them by months. Balance assumption**:** $\sum_{j=1}^{n} m_{ij} x_j \leq M_i$, where m_{ij} is the specific labor requirement of the j^th plant during the i^th period (in working hours), while M_i is the available capacity expressed in working hours.

  When we are making a plan, it is also important to consider the fact that in peak periods, besides the permanent staff, seasonal workers, who are usually working with a fixed-term contract, are also needed. In this case, labor is a non-fixed capacity but as a flexible bound, and the number of employees is optimized jointly with the production structure: $\sum_{j=1}^{n} m_{ij} x_j \leq M_i + M_i^{'} \beta$, where $M_i^{'}$: the number of working days worked by one employee during the given i^th^ period, while β means the number of employees. The final solution of the model provides the required number of workers in peak period.

- Balance assumption of current assets necessities: $\sum_{j=1}^{n} e_{ij} x_j \leq E_i$, where e_{ij}: specific currents assets necessities of j^{th} activity during the i^{th} period, while E_i is the current assets capacities available in the i^{th} period.
  The available capacity should not be less than the total current assets necessities. The unit is one thousand Forints on both sides.
- Machine balances are to be calculated by the types of machines and categories of machines per decades. Balance assumption: $\sum_{j=1}^{n} g_{ij}^{h} x_j \leq d_i^h$, where d_i^h: the number of shift days that can be performed by the h-type machine during the i^{th} period (i.e. the available capacity), g_{ij}^{h} the specific machine demand of the j^{th} activity in shift days of the h-type machine in the i^{th} period.

Based on balance assumptions, the linear programming model looks like the one shown in Table 1.

Table 1: Planning arable crops production for one year with the simultaneous optimization of resources and crop ratio

	Plant $_1$	…	Plant $_n$	Consumption	Relation	Capacity
Crop rotation	Technology matrix			Quantity consumed =SUMPRODUCT function	=	Available resources (number value)
Powertrain capacity					< =	
Other limiting factors						
Objective function	gross margin per hectare			GOAL (MAX gross margin)		
Solution (hectare)	area per plant					

Source: Own editing

1.3.3. The multi-periodic crop ratio model

The multi-periodic linear programming model is for several years**.** For its establishment, the first step is to make a technology matrix for one year, then the certain technologies should be interlaced (diagonal hyper matrix) with the help of the so-called transfer variables. I placed annual crop production models to the diagonal of the matrix and zero values to the other elements of the matrix (Table 1). When we establish a model, the following aspects should be taken into consideration: a model should reflect reality as much as possible; it is also important that it should be easy-to-use from a mathematical point of view and also from an IT point of view. The LP model could be easily run in Microsoft Excel.

Table 1: The basic pattern of multi-periodic linear programming model

			1st year									…	'i' year									…	'n' year									R	C
			VC			EP			M_i	M_g		VC			EP			M_i	M_g		VC			EP			M_i	M_g		
			VC_1	….	VC_n	EP_1	…	Ep_n				VC_1	….	VC_n	EP_1	…	Ep_n				VC_1	….	VC_n	EP_1	…	Ep_n				
1st year	VC	T_m																													<=	F_i
		V_m																													<=	F_i
		M_m							-1																						<=	M_i
		SGM_m								-1																					<=	d_i^h
		GM_m																													<=	d_{iv}^h
	EP	T_{EP} 2nd year				-1									1																=	0
		…					-1									1															=	0
		T_{EP} 2nd year						-1									1														=	0
⋮											⋮																				⋮	⋮
'i' year	VC	T_m																													<=	F_i
		V_m																													<=	F_i
		M_m																-1													<=	M_i
		SGM_m																	-1												<=	d_i^h
		GM_m																													<=	d_{iv}^h
	EP	T_{EP} i+1.év													-1									1							=	0
		…														-1									1						=	0
		T_{EP} i+1.év															-1									1					=	0
⋮																					⋮										⋮	⋮
'n' year	VC	T_m																													<=	F_i
		V_m																													<=	F_i
		M_m																											-1		<=	M_i
		SGM_m																												-1	<=	d_i^h
		GM_m																													<=	d_{iv}^h
CF			MC						BF		…	MC						BF		…	MC							0	MAX	MC!
S			PS						BM		…	PS						BM		…	TSZ							BM		

Where:

VC	: Vegetable cultivation variable n=1,2…,k	GM_m	: Machine work is a balance sheet condition
EP	: Energy plantation variens n=1,2…,k	T_{EP}_n ye	: Settlement transfer condition n=1,2…,k
M_i	: Temporary work (work hours)	CF	: Objective Function
M_g	: Machine work (work hours)	S	: Solution
T	: Transfer variable	PS	: Production structure (100 hectares)
BF	: Specific cost of rental work	R	: Relation (=;<=;>=)
BM	: Amount of rental work (work hours)	CF	: Vector capacity
E	: Resources	F_i	: Available land (100 hectares)
T_m	: Balance sheet for the area	M_i	: Provide workforce (work hours)
V_m	: Conversion balance requirement	d_i^h	: Available for work (work hours)
M_m	: Balance sheet for labor force	d_{iv}^h	: Machine work available (work hours)
SGM_m	: Balance sheet for own machine work	MC	: Marginal contribution (one thousand HUF/100 hectares)

Source: Own compilation

In the basic pattern of the model, the signs for each parameter are represented by the following symbols:

negative sign

positive sign

negative and positive sign

A reference farm with average parameters could be easily defined with more than 100 variables and a few 1000 balance assumptions. Balance assumptions should

be formulated on the basis of practical experience on acreage, crop rotation, maximum size of plantation, skilled and unskilled labor and machinery.
It is preferable to interlace the consecutive years of the multi-periodic linear programming model with individual balance assumptions:

- If we compete only arable crops with each other, we should use it to take the green crops into account
- If we intend to plant woody energy crops into one of the examined reference farms, several aspects should be taken into consideration:
    - If the plantation has become a part of the production structure in the first year, from the following year onwards, the size of the acreage has to be the same or even bigger
    - If the plantation was not included in the production structure in the first year, then in the second year, it should be taken into account that it might be a part of the production structure from that year. In this case, delayed harvest periods should be taken into consideration.

To make modelling easier, I have come up with a special balance assumption; as the plantation remains in the same area for 15-20 years, depending on its species, it occupies the area. Therefore, calculating with the crop rotation limits can be ignored. In the model, this bound can be solved by introducing the following balance assumption:

$$x_j^k = x_j^{k+1}$$
$$\downarrow$$
$$x_j^k - x_j^{k+1} = 0$$

$, \text{ahol}$, where $k = 1, 2, ..., n$: number of the current year, x_j^k is the given year, while x_j^{k+1} is the year, following the given year.

2. Summary

By writing my article, my aim was to develop a summary about how to optimize production structure nowadays. Since linear programming has been used in several areas for decades, it is also possible to optimize crop production with it. Since nowadays we would like to produce raw materials for food and for forage in less-favoured areas, I have developed a multi-periodic linear programming model. By uploading the model with basic data, sensitivity analysis is also possible, from which, information could be gained on resources and activities.

Bio-note

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