MODELING ACTIVITIES OF COMMERCIAL BANK THROUGH PETRI NETS

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Abstract: *The relevance of the article is determined by the need to improve the methods of modelling and simulating commercial bank activity, including for the purpose of calculating, controlling and managing the risk of the bank, in the context of the transition to the application of Basel III standards. This improvement becomes necessary because with a direct transition to new regulatory standards internal assessments of the main risks become the initial data for calculating the capital adequacy of a bank. We believe that commercial banks are interested in developing existing practices of modelling and simulating activities, including risk management purposes. In other words, we must improve the formalization of logical and mathematical representation of connections, regularities and the explanatory theory for the operations in commercial bank. We believe that attaining similitude between commercial bank and its model by Petri nets has a number of advantages compared to other instruments. This involves some opportunities of continuous improvement of the theoretical model, the possibility of slight changes of the degree of homomorphism, the possibility of concomitantly including variables typical of deterministic models, stochastic models, as well as fuzzy (vague) models. It is important that Petri nets have a graphic representation with a particularly effective impact on intuitively understanding the systems dynamics. The article reveals the possibility of modelling the activity of a financial institution through Timed Hybrid Petri Nets. The purpose of this article is to study the possibility of using Petri nets for modelling activities of a financial institution. The present study involved the employment of universal methods of economic analysis, namely, the method of scientific abstraction, mathematical modelling and the method of functional analysis. The content of the article refers mostly to summarizing the extensions of Petri nets that can be used to model, review and evaluate numerical characteristics of financial institutions' performance. This article deals with generalized timed Petri nets, timed continuous Petri nets, and generalized timed hybrid Petri nets. The main result obtained in the study and presented in the article is the argumentation of the possibility to analyze the quantitative and qualitative characteristics of a commercial bank with the help of Petri net extensions.*

Keywords: *commercial bank; modelling; Petri nets; qualitative properties; quantitative properties; financial institution; generalized timed Petri nets; timed continuous Petri nets; generalized timed hybrid Petri nets.*

JEL Classification: *G17; G21; G23; C02; C60.*

1. Introduction.

For the purpose of modelling, a financial institution can be regarded as a subsystem of the economic system. Thus, the internal parameters for the system are external for the financial institution and we will have to develop a model that describes the interdependence between input and output parameters of the subsystem. The size and the uncertainty of output subsystem parameters are dependent on the corresponding parameters of input.
In this paper, we present an approach to modelling activities of a financial institution that has as its starting point the fact that the functions of parallel/distributed application are achieved through *cooperative actions* of data processing. In this context, cooperation implies the capability of the processes to communicate and the correlation of the actions with a focus on achieving common tasks.
Formulating *a mathematical model* for a *real system* is crucial and much more difficult than solving it and interpreting its results. In this regard, one of the possibilities is the use of Petri nets (Aalst W.M.P., 1998, Juncan T., Ţiplea F. L., 1995, Li Hui- Fang, Fan Yu-Shun, 2004).
Hereafter, we will briefly present some extensions of Petri nets that can be used in modelling, verification and evaluation of numerical characteristics of financial institutions' performance.

2. Generalized Timed Petri Nets.

The theory of autonomous Petri nets relates to the order of events and, consequently, the net dynamics is regarded as a sequence of events (transitions) restricted to considerations of logic (a transition can start only if enabled). In this context, the question "how long does an event take?" is not asked. Yet, in order to answer questions related to the net performance and a modelled system, it is necessary to take the time into account. The time (duration) may be related to places and transitions in the following way:

- Time related to transitions: the time separating the start of the action (the consumption of the tokens in pre-places) and completion of the action (the production of tokens in post-places) corresponding to the transitions. These periods of time are named for action time.
- Time related to places: minimum periods of time for the tokens to become constant in a place before being able to contribute to the enabling (and firing) of the following transition. These periods of time are called delay time.
- Duration of transition firing can be used, for example, to represent the transfer time in banking systems (where the transition means time used for a transaction). Action times, delay times and firing times can be constant or variable, can be deterministic or nondeterministic.
  From the point of view of modelling capability, these two types of delays are equivalent (Alla H, David R, 1992).

Hereafter, we will use the timed Petri nets in which time is related to transitions, as the specifics of the analyzed processes is determined by the parallel sequence of events and concurrent actions.
The modification of the original model of Petri nets involves not only the simplification of their representation but also their extension. Petri nets extensions correspond to some models, where there have been added additional rules to allow treatment of a

broader range of applications and designs. In some cases, this became possible by extending syntax, without significant deviation from basic computing models - the Turing machine. The most significant models that extend the descriptive power of a basic computing model (also called autonomous models) are timed Petri nets, stochastic Petri nets, continuous Petri nets and hybrid Petri nets (Alla H, David R, 1992, Enicov I., Guţuleac E., 2007, Molloy M. K., 1982).
Further on we present a variant of Generalized Timed Petri Net(GTPN) subjacent to the timed hybrid Petri nets, (Guţuleac E., 2004, Guţuleac E., Reilean A., Boşneaga C., 2002), which can be used to model and evaluate the risk of financial institutions' performance.
Definition 1. A Generalized Timed Petri Net, in abbreviated form GTPN, is a structure expressed in terms of 11-tuple: *RPG=<P, T, Pre, Post, Inh, Test,* K_p, *Pri, G,* τ , M_0 >, where:

- P is a finite set of *places,* $|P| = k$;
- T is a finite set of *transitions*, $|T| = n$ and P□T=□;
- *Pre*, *Inh* and *Test* : PxT□ μP are the functions of the arcs weight: *Pre* is the incidence function of pre-places (direct arcs of transition input places of a transition), *Inh* is the inhibition function of transitions by means of inhibitor arcs, whereas *Test* is the function that describes the possible loops of an impure network, meaning *Pre*(*p,t*) = *Post*(*p,t*). An *arc test* is represented by dashed arcs. *Post*: TxP□ μP is the incidence function of post-places (direct arcs of output places of a transition). These functions can be marking-dependent, i.e. they are determined by a number of multisets μP of P;

- $K_p : P \rightarrow IN \cup \infty$ is the function of the place capacity, it is considered unlimited by default. *IN* is a set of non-negative integers;
- *Pri* : T□IN is the priority function of a transition (it is considered null by default);
- G : T x $IN^{|P|}$ □{*true, false*} is a *switching* function which determines a Boolean function $g(t, M)$ for each t in this net marking. If function $g(t , M)$ is '*true*' and t is enabled by the current marking M, then this transition remains enabled and eventually it can be fired; yet, if $g(t , M)$ is '*false*', then it can not be fired, having function $g(t , M)$ is '*true*' by default;
- $M : P$□□N , $M = [m_1,\ldots,m_i,\ldots,m_k]$ with $m_i = M(p_i)$ is the function of marking places, and M_0 is the initial marking of the net;
- τ : $T \rightarrow IR_+$ is the time needed for firing a transition enabled by current marking. IR_+ is a set of non-negative real numbers. Transitions $t_j \in T(M)$ enabled by current marking M , which were selected for the firing process, will change this marking after time τ_j .

The weight of different types of net arcs, which are not explicitly mentioned, is considered to be 1. The capacity of a place is considered unlimited by default. Also, if firing priorities and switching functions of transitions t_j are not explicitly mentioned, then it will be considered that they have the following values: $Pri(t_j) = 0$ and $g(t_j , M)$ ='*true*'.
In what follows, we will use the following notations:

- ${}^{\bullet}t_j$ ={$p_i \in P$: *Pre*(p_i, t_j)>0 } and $t_j^{\bullet}$ ={$p_i \in P_d$: *Post*(t_j, p_i)>0} is a multiset of input places and output places of transition t_j, respectively;

- ${}^{\circ}t_j$ ={$p_i \in P$: *Inh*(p_i,t_j)} and ${}^{*}t_j$ ={$p_i \in P$: *Test*(p_i , t_j)} is a multiset of control places through inhibitors arcs and transition test t_j, respectively;

Definition 2. (*Rule of firing an enabled transition*). A transition t_j of *GPN* is *enabled* by current marking M, noted as $t \in T(M)$, only on condition that, irrespective of its priority, it involves the verification of the following Boolean function, which is subject to verification $cs(t_j, M)$: $ec(t_j, M) = (\wedge_{\forall p_i \in {}^{\bullet}t_j} (m_i \geq Pre(p_i,t_j)))$ & $(\wedge_{\forall p_i \in {}^{*}t_j} (m_i \geq Test(p_i,t_j)))$ & $(\wedge_{\forall p_i \in {}^{o}t_j} (m_i < Inh(p_i,t_j)))$&(g($t_j$, M) = 1) & $(\wedge_{\forall p_n \in e_j^{\bullet}}((K_p - m_n) \geq Post(p_n, e_j))$

Transition t_j $T(M)$ is enabled after time τ_j, on condition that there *is no* other transition t_k of a higher priority *Pri*(t_j) > *Pri*(t_k) *for which* the conditions of enabling t_k $T(M)$ and timing are verified. Firing transition t_k from current marking *M* brings to a new marking $M' = M - Pre(\bullet,t_j) + Post(\bullet,t_j)$, wherre $Pre(\bullet,t_j)$, $Post(\bullet,t_j)$ are the functions induced by *Pre*, *Post* to *P*. The fact of firing transition t_j from current marking *M* is noted as $M[t_j > M'$.

3. Timed Continuous Petri Nets.

Timed Continuous Petri Nets were introduced by H. Alla and R. David (Alla H, David R, 1992, Alla A., David H., 1998). The particularities of these types of nets are in the fact that the marking of a place is a positive real number and not an integer. The firing of the transition is carried out by a continuous flow of fluid. This type of nets allows the modeling of the systems that can not be modeled by discrete Petri nets and leads to the creation of a new model, approached in a convenient way, when the number of reachable tokens of a discrete Petri net becomes too high, which will be a limit to the use of Petri nets for modeling such processes.

A *Timed Continuous Petri Net* is an extension of Petri nets, in which a number of tokens in their places and the corresponding incidence functions are defined as real numbers.

Next, we present an extension of Timed Continuous Petri nets (TCPN).

Definition 3. T-timed Continuous Petri Net, in abbreviated form (TCPN), is 11-double:

RCT = <*P, T, Pre, Post, Inh, Test,* K_p, *Pri, G, V,* M_0 >,

Where the definitions of *P*, *T*, *Pre*, *Post*, *Inh*, *Test*, *Pri,* K_p, *Pri, G,* are similar to the ones presented in the discreet Petri nets, apart from the fact that the incidence functions of *Pre, Post, Inh* and *Test* are real numbers;

- *V* is an application of a set of transitions *T* of a TCPN to a set of real numbers $R^+ \cup \{\infty\}$. Speed $V(t_j) = V_j$ corresponds to the maximal speed of transition firing t_j;

- M_0 is the initial marking of this net which is rendered by a vector of positive real numbers or null, which can also be noted as $\vec{x}_0 = X(0)$, knowing that $\vec{x}(\tau) = (x_1(\tau), \cdots, x_n(\tau))$, $n = |P|$ denotes the current marking of TCPN at time τ.

A place p_i having null as marking can enable a transition $t_j \in p_i^{\bullet}$. In order to obtain it, it is enough to have a place fed by a transition $t_k \in {}^{\bullet}p_i$ at the input of this place.

Definition 4. A location p_i is fed at time τ, if and only if there is at least a transition t_j in ${}^{\bullet}p_i$ which is enabled.

A transition t_j *is enabled* at time τ, if all the places $p_i \in {}^{\bullet}t_j$ meet at least one of the following two requirements: (i) $m_i(\tau) > 0$; (ii) p_i is fed.

We assume that the transition is *strongly enabled* if all the places from set ${}^{\bullet}t_j$ meet the first requirement. And, it is *weakly enabled* if it does not meet the first requirement.

In a TCPN, a transition t_j, whose real firing speed is $v_j(\tau) > 0$, is fired continuously. The marking of a place p_i, at time $\tau + d\tau$ is deduced from the current marking at time τ, using the following relation: $m_i(\tau + d\tau) = m_i(\tau) + \sum_{k=1}^{m} [Post(p_i, t_k) - \Pr e(p_i, t_k)] \cdot v_k(\tau) d\tau$,

which brings us to the following *fundamental relation*: $\frac{dM(\tau)}{d\tau} = C \cdot v(\tau)$,

where *C* is the incidence matrix of TCPN, whereas $v(\tau)$ is the corresponding firing speed vector of a transition at time τ, i.e. $v(\tau) = (v_1(\tau), ..., v_m(\tau))$.

Based on the fundamental relation, it is possible to verify different interesting properties of TCPN. Some of them, such as place invariants or transition invariants, are similar to the ones of the discreet Petri nets. Others, such as operation intervals and evolution graphs for TCPN are typical of a continuous model (Alla H, David R, 1992, Alla A., David H., 1998).

TCPNs may approximate a system of discreet events. At the same time, they also allow to model systems with continuous flows of data (leakage and/or, for example, flow blending) without being discretized. Moreover, they allow to model systems, in which parameters that characterize the nature of these flows can be very heterogeneous, for example, banking transactions, which will consider requests or offers (integers by nature) and sums of money (rational numbers).

TCPNs are well suited for modelling permanent operations with continuous flows by nature. Yet, in a financial system, operation processes are discreet-continuous by nature and there can often happen operation failure, when one or more resources are not available and then corresponding maximal speed *V* becomes invalid.

Suchlike situations can be modelled through *Generalized Timed Hybrid Petri Nets*, in abbreviated form *GTHPN*s, which will contain *continuous places and transitions* (hereinafter *C-places* and *C-transitions*) and *discreet places* and *transitions* (hereinafter *D-places* and *D-transitions*).
Further on, we will consider an extension which is called *Timed Hybrid Petri Nets*.

4. Generalized Timed Hybrid Petri Nets

To make a formal representation of a *GTHPN* we will again consider an autonomous discreet Petri net. We assume that transition t_j and place p_i are the corresponding transition and place of this net. Marking of place p_i is an integer of tokens in this place and is noted as m_i. We will also analyse a similar net model, though it is built up in a way that a set of places and transitions are divided into two disjointed parts: each place and each transition can be either discreet or continuous. The obtained net is called a Hybrid Petri Net, in which the marking of continuous places is a real number, while the marking of discreet places is an integer. If all the nodes of the net are discreet, then *GTHPN* is transformed into a timed discreet *Petri net.* If all the nodes of the net are discreet, then GTHPN is transformed into a *TCPN*.
Definition 5. A Generalized Timed Hybrid Petri Net, in abbreviated form GTHPN, is a quadruple $RHGT = < RPG,\ h, M_0, \vec{x}_0 >$ which conforms the following conditions:

1) A GPN is a generalized petri net, in which the set of places P makes up a partition $P = P_d \cup P_c$, $P_d \cap P_c = \varnothing$, where P_d is the set of discreet *D*-places and P_c is the set of continuous *C*-places, while the set of transitions *T* makes up a partition $T = T_d \cup T_c$, $T_d \cap T_c = \varnothing$, where T_d is a set of discreet *D*-transitions and T_c is the set of continuous *C*-transitions. The set of arcs *A*, determined by the incidence function *Pre, Inh,* $Test: P \times T \to \mu P$ and $Post: T \times P \to \mu P$ also make up a partition $A = A_d \cup A_c$, $A_d \cap A_c = \varnothing$, so that $A_d = (P_d \times T) \cup (T \times P_d)$ and $A_c = (P_c \times T_c) \cup (T_c \times P_c)$;
2) application $h: P \cup T \to \{D,\ C\}$, called hybrid function, indicates the type of every node of the net, whether it is discreet *D* or continuous *C*. When $p_i \in P_d$ is a *D*-place, then the corresponding incidence functions *Pre*(p_i, t_j) and $Post(p_i, t_j)$, $Inh(p_i, t_j)$ and $Test(p_i, t_j)$, $\forall t_j \in T$ are non-negative integers; yet, if $p_i \in P_c$ is a *C*-place, then $\forall t_j \in T_c$ these functions are positive real numbers.
3) for $\forall h(p_i) = D$ and $\forall h(t_j) = C$ *Pre* incidence functions and *Post* incidence functions verify the following relation *Pre*$(p_i, t_j) = Post(p_i, t_j)$;

4) current marking $(M, \vec{x})$ of the net is determined by the vector of the number of tokens $M = (m_i, p_i \in P_d)$ in *D*-places and the vector of continuous marking $\vec{x} = (x_k, p_k \in P_c)$ of *C*-places. In the initial state the net has initial marking $(M_0, \vec{x}_0)$.

An incidence matrix *W* corresponds to each of the hybrid Petri net:

$$W = [W_{ij}]_{n \times m} \text{ , where } W_{ij} = Post(p_i, t_j) - Pre(p_i, t_j).$$

The third requirement from definition 4.15 relates that an arc must connect a C-transition with a *D*-place, on condition that there is a reciprocal arc, which allows to ensure that the marking of *D*-places must always be an integer whatever the evolution of marking in the net is.
The marking of *C*-places is represented by a real number, an amount of fluid, whereas the marking of *D*-places is, as usual, represented by black dots, tokens by name.
Hereafter in order to distinguish continuous transitions and places from the discreet ones, we will mark continuous transitions as $u_k \in T_C$ and continuous places as $b_k \in P_C$.

Rules of enabling and firing of transitions. A discreet transition $t_j \in T_D(M)$ is enabled by the current marking *M* if the following logical expression (enabling condition $ec_d(t_j)$) is verified.

$$\begin{gathered} ec_d(t_j) = (\underset{\forall p_i \in {}^\bullet t_j}{\wedge} (m_i \geq Pre(p_i, t_j))\,\&\,(\underset{\forall p_k \in {}^\circ t_j}{\wedge} (m_k < Inh(p_k, t_j))\,\& \\ (\underset{\forall p_l \in {}^* t_j}{\wedge} (m_l \geq Test(p_l, t_j))\,\&\,(\underset{\forall p_n \in t_j^\bullet}{\wedge} ((K_p - m_n) \geq Post(p_n, t_j))\,\& \\ (\underset{\forall b_i \in {}^\bullet t_j}{\wedge} (x_i \geq Pre(b_i, t_j))\,\&\,(\underset{\forall b_k \in {}^\circ t_j}{\wedge} (x_k < Inh(b_k, t_j))\,\& \\ (\underset{\forall b_l \in {}^* t_j}{\wedge} (x_l \geq Test(b_l, t_j))\,\&\,(\underset{\forall b_n \in t_j^\bullet}{\wedge} ((K_b - x_n) \geq Post(x_n, t_j))\,\&\,g(t_j, M) \ . \end{gathered}$$

Also, a continuous transition $u_j \in T_C(M)$ is enabled by the current marking M if the following logical expression (enabling condition $ec_c(u_j)$) is verified.

$$\begin{gathered} ec_c(u_j) = (\underset{\forall b_i \in {}^\bullet u_j}{\wedge} (x_i > 0)\,\&\,(\underset{\forall p_k \in {}^\circ u_j}{\wedge} (m_k < Inh(p_k, u_j))\,\& \\ (\underset{\forall p_l \in {}^* u_j}{\wedge} (m_l \geq Test(p_l, u_j))\,\&\,(\underset{\forall b_k \in {}^\circ u_j}{\wedge} (x_k < Inh(b_k, u_j))\,\& \\ (\underset{\forall b_l \in {}^* u_j}{\wedge} (x_l \geq Test(b_l, u_j))\,\&\,(\underset{\forall b_n \in u_j^\bullet}{\wedge} ((K_b - x_n) \geq V_j \cdot Post(x_n, u_j))\,\& \\ g(t_j, M) \end{gathered}$$

An enabled transition can be fired. The firing of a D-transition t_j consists of removing *Pre*(p_i, t_j) tokens from each input place p_i to transition t_j and adding

$Post(p_i, t_j)$ tokens to each output place of the transition fired. Firing of quantity *r* of a *C*-transition u_j involves removing $r \cdot Pre(p_i, u_j)$ of the fluid from each input place b_i to transition u_j and adding $r \cdot Post(p_i, t_j)$ fluid in each output place b_j out of u_j.

An enabled discreet transition $t_j \in T_D(M)$ (continuous $u_j \in T_C(M)$) will be *enabled* if there is no other transition $t_k \in T_D(M)$ ($u_k \in T_C(M)$) enabled, which has a priority over it.

Let's assume that σ is a firing sequence of transitions and $\overline{\sigma}$ is the characteristic related to σ. The size of the vector $\overline{\sigma}$ is equal to the number of net transitions *k*. Component $\overline{\sigma}_j$ of vector $\overline{\sigma}$ is represented by the number of transition firings t_j in a firing sequence σ and this number is noted as $N_j = N(t_j)$. If t_j is a *D*-transition, then N_j is an integer, whereas if t_j is a *C*-transition, then N_j is a real number.

Then one can determine the reachable marking *M* out of initial marking *M* through a firing sequence σ, $M_0[\sigma > M$, using the following fundamental relation (Alla H, David R, 1992):

$$M = M + W \cdot \overline{\sigma}$$

The fundamental relation of a Hybrid Petri net is identical to the one of a discreet Petri net. Thus we can anticipate that all the properties of discreet Petri nets which are intrinsic to this relation can be referred to the Hybrid Petri nets as well.

Definition 6. A Generalized Timed Hybrid Petri net is *double* $RHGT = < RHG, \theta, V >$, where: *A HPN* is a marked hybrid Petri net, specified in accordance with definition 5, in which a set of discreet transitions T_D is distributed in the following way $T_D = T_\tau \cup T_0$, $T_D = T_\tau \cap T_0 = \varnothing$:

- T_τ is a set of timed transitions with a finite firing time;
- T_0 is a set of immediate transitions with a null firing time;

θ:*T*µ*P IR*$^+$ is the function that determines a firing time parameter of an enabled discreet transition *tT*(*M*): if *t* is a timed transition, then $\theta(t, M) = d(t, M)$ is the transition firing time of $t \in T_\tau(M)$ in current marking *M*;

- if t' is an immediate transition, then $\theta(t', M)$ is the weight of this transition which determines the probability of the firing *a*(t', *M*) = $\theta(t, M) / \sum_{t_k \in T_0(M)} \theta(t_k, M)$ of a transition $t \in T_0(M)$ in current marking *M*, which describes a probabilistic selection of free choice of the transitions in a structural conflict $t' \in {}^{\bullet}p$ regarding place *p*, so that $0 \leq a(t', M) \leq 1$, $\sum_{(t' \in {}^{\bullet}p)} a(t', M) = 1$;

- $V : T_C \times \mu P \to IR_+$ is the function that determines the maximal speed of firing related to a continuous *C*-transition u_j, so that the level of the fluid of a continuous place x_j will continuously change.

- μP is a multiset of places *P*, and IR^+ is a multiset of non-negative real numbers.

The state of GTHPNs is defined by its marking. For a Timed Petri net (whether hybrid or not) current marking M is decomposed into $M = M^r + M^n$, where M^r and M^n are *reserved marking* and *non-reserved marking* of a net, respectively. Only non-reserved marking M^n is taken into consideration in order to enable a transition. Moreover, C- transition firing speeds are deduced from this non-reserved marking. The state of a hybrid Petri net has no duration as C-places marking continuously varies. However, there is a time interval when D-places marking and C-transition firing speeds remain constant.

unde $M(\tau)$ este marcajul curent la momentul τ, iari *W* este matricea sa de incidenţă.

A set of reachable marking of a timed Hybrid Petri net is included in the set of reachable markings of an autonomous hybrid Petri net subjacent to this timed net. Current marking $M(\tau)$ at time τ is deduced from initial marking *M*(0), using the following fundamental relation:

$$M(\tau) = M(0) + W(\overline{\sigma}(\tau)) + \int_{u=0}^{\tau} v(u)du),$$

where $M(\tau)$ is a current marking at time τ, and *W* is an incidence matrix.

In this relation, characteristic vector $\overline{\sigma}(\tau)$ represents a number of firings of each D-transitions (discreet interpretation) between initial time $\tau_0 = 0$ and current time τ. C-transition components are equal to zero. Speed vector components $v(\tau)$ represent instantaneous firing speeds of C-transitions at time τ. D-transition components are equal to zero. This relation separates discreet evolution from continuous evolution. It is a trajectory in discreet-continuous space of a timed Hybrid Petri net.

5. Conclusions

Designed to model distributed systems, where concurrence and parallelism are central, Petri nets of various extensions have promptly become the reference model to describe these types of systems. Their application in the field of engineering has propelled them into the spotlight of researchers.

Autonomous Petri nets and their extensions are also of the utmost interest thanks to the clarity of representation of a control flow in a system of interdependent activities.

At the same time, the theory of Petri nets allows rigorous demonstrations of the systems behavior described by this formalism, observing some interesting properties in terms of cooperation of concurrent processes: mutual exclusion, synchronization, etc. Based on behavioural properties of Petri nets, it is possible to identify, for example, the correctness of the appropriate operating structures in relation to the specified restrictions.
Petri nets extensions are used to model, validate running processes and evaluate the system performance as well as parallel/distributed applications. Once a model has been developed for the given system, it is possible to perform a qualitative analysis of the functioning of this system.
Such quantitative and qualitative properties of commercial bank activity can be explored through Petri nets extensions, ensuring a relative simplicity of formulating logical and mathematical representation of connections, regularities and explanatory theory for processes of commercial bank and, simultaneously, providing graphic support with a highly efficient impact on understanding the dynamics of the model intuitively.

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