APPLYING GAME THEORY IN RISK MANAGEMENT

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Abstract: This article is devoted to analysis methods of application of game theory in conflict situations or competition in the economy. There are examples of the use of game theory in the identification, sizing, analysis and risk management. There is described the defining concepts of game theory in terms of their application in risk management. It is shown how to use the decision theory in risk management, which is part of operational research and decision-making process, based on various mathematical models. The decision theory refers performing various actions, to achieve well-defined goals in uncertain circumstances and under risk situations. The quality of decision is subject to a complex set of preconditions, it's seen determines by the quality of used information and held managerial skills and applied by decision makers. The Choice between different alternatives of optimal decision is based on economic calculations of rationality, but also filled with other criteria that are not necessarily related to the economy. It thus becomes important to establish a set of rules and criteria for evaluating the performance of decision. This article presents games against nature (economic circumstances will be considered the nature). In the situations when a player (trader) wants to make an investment or a decision on an economic business etc., which conflicts with nature (incidentally nature of economic events), these decisions involve taking a certain risk and / or uncertainty regarding developments in the future. The player may reduce the risk of taking wrong decisions (for lack states of nature) by doing more experiences. This is however limited by the following circumstances: experiences requires time, in most cases, the decision must be made quickly and on the other hand, experiences involves expenditure which may be greater than the benefit brought by the excess of information that experiences provide. Thus, depending on the number of experiences that player decides to perform, in the article are analyzed: statistical inexperienced games; statistical games with unique experience; sequential statistical sampling Games; statistical sampling sequentially truncated. Games.

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Game theory. General considerations.

Game theory is a chapter of the *operational research* methods and aims at determining the choice of the best decisions in *conflict situations* in many rational factors pursuing opposing interests.

John von Neumann and Oscar Morgenstern (1944) are the first authors who analyzed based game theory and conflict situations in the economic field, where is the presence of a system of free competition, participants with opposing interests are companies, trade houses etc. Conflict in the economy is also when a company conflict with incidentally character of natural events (economic situation) called *conflict with nature*.

Game theory can be defined as a mathematical theory of conflict situations.

By *game* means a situation where it acts a lot of rational elements (called players, partners or adversaries) which sequentially and independently following a set of rules, choose one *decision* (takes one move) a lot once alternatives. The rules are specifying the conditions under which the game is over and reward (or gain) that each player receives every situation.

The games that will have a conscious action plan of players, a strategy that they must prepare and follow , will be called a strategic game .

In game theory uses the concept of pure strategy. If in play, one of the opponents has available *m* alternative and match (game) ends with a choice, then it is said that the player has *m* pure strategies. If games against nature, each strategy corresponds to a state of nature.

Each player aims to apply a strategy to bring maximum gain therefore seeks an optimal strategy.

In terms of earnings, the games are classified into zero-sum games when in the end of a game the winnings of a player is the lost of other player, and no zero-sum games, when part of the sums lost by a player (or more players) are reserved for purposes other than to pay the winner player (or players).

The number of pure strategy devide games into finite games, in which each player has a finite number of pure strategies, and infinite games when the number of strategies are infinite.

Game theory seeks to develop recommendations for a logical partners reaction, providing certain guarantees to players if they have chosen a correct behavior.

The principle of a player is the principle of non-risk situation: a player must choose behavior taking into account the most unfavorable action off the opponent.

In reality there are situations where the risks to take the decision may not be known, even in probability, and this is a situation of total uncertainty. Decision theory analyse such situations.

Decision theory. General considerations

Decision theory is a chapter of operational research and decision-making study based on various mathematical models. Decision theory refers perform various actions so as to achieve objectives (goals) well defined in uncertain circumstances, uncertainty, under risk. It is an evolved version of game theory.

The basic concept being operated in decision theory is the decision. By decision means a process by which an intelligent system called decider (specialist, dedicated software, etc.), determine the relevance of certain changes in behavior and develop possible alternatives in this regard, choosing one of them as adopted course of action for achieving the goal . As a result, the decision appears as a character invested with the information directory.

The decision process whose duration is limited in time and constraining for decider is structured, from a procedural standpoint, the following steps:

- Diagnosis and structuring the problem / situation decision
- Design and evaluation of possible alternatives
- Selection and adoption rate preferred share

Going through these phases is not strictly sequential, but involve any reconsideration of results or resumption of iterations.

In the initial stage of its assertion that discipline, decision theory concentrated on algorithmic and computer invoice approach to decision-making procedures, reflected mainly in statistical and mathematical models of decision and operational research. Recent developments of decision theory, based on a systems approach and brought into line with practice data reveals ever more salient character deliberative, qualitative judgment, decision-making; they concluded that the rationality of decision making is reflected largely in the application of procedures heuristic (deciding not based on the best solution is usually quite difficult to find, but on the best solution found until decision), in recognition of the widespread application of intuition and emotional states.

The quality of decision is subject to a complex set of preconditions, determining the quality of information used and held managerial skills and applied by decision makers. The growing complexity of decision-making in contemporary organizations determine the development, within their capacities for decision preparation by advisors and experts, including assistance with computer equipped with a dedicated software in generating appropriate solutions to solve the problem. Choosing between different alternatives of optimal decision is based on calculations of economic rationality, but also filled with other criteria that are not necessarily related to the economy. It thus becomes important to establish a set of rules and criteria for performance evaluation decision.

Games against nature

Next we'll look at the games against nature (such as economic circumstances will be considered nature, given the random and uncontrollable thereof by the player), ie situations when a player (trader, bank, specialized institution, etc.) wants to make an investment, a decision on an economic business etc., which conflicts with nature (nature chance of economic events) that decisions that involve taking a certain risk , and / or some uncertainty regarding developments in future.

In these games, it is considered nature to take its most unfavorable for the opponent (player). Therefore, the player may not be in complete ignorance ,has most often able to obtain some information on the strategies of nature through experiments (research, analyzes, studies, etc.) you can do and to give them the opportunity to maximize the return on the nature and reserve it. In these cases the games due to statistical experiments are carried out, call statistics and mathematics is due to their theory Alexander Wald (1951).

The player may reduce the risk of taking wrong decisions (for lack states of nature) by experience. This is however limited by the following circumstances: experiences requires time, in most cases, the decision must be made quickly and on the other hand, experiences involving expenditure which may be greater than the benefit brought by the excess of information they provide experiences .

In those games, an important issue is the decision on conducting experiments ie: if you have made experiences and, if necessary - on which one to do, when to stop and what actions undertaken with the experiences end.

Thus, depending on the number of experiences that the player decides to perform statistical games are as follows:

- Inexperienced statistical Games
- Statistical games with unique experience
- Statistical sampling sequentially Games
- Statistical sampling sequentially truncated Games

Statistical inexperienced Games

By pure nature strategy we understand the complete assembly of external conditions present in decision-making, ie an actual state of nature. Generally, there are a lot of possible states of nature, which we note:

$\Theta=\{\theta_1, \theta_2, \theta_3, \ldots, \theta_m,\}$

where *m* is the number of pure strategies of nature. This set will be called the space of states of nature or nature's pure strategies.

From past experience it is known how often adopt its nature a pure strategy or another, that is known a priori probability distribution of ξ (θ) on the state space Θ nature. This distribution will be called mixed strategy of nature.

Let $a_1, a_2, \ldots a_n$ be the action that statistician-player has.

The crowd:

$A=\{a_1, a_2, \ldots a_n\}$

Space for pure strategies of statistician-player.

The function L (θ, a) will function gain and the gain will be that you will get the player taking the action when nature is in state θ. This function must be defined a priori for all of the A and θ of Θ, ie the direct product of the sets $\Theta \times A$. It can be given in matrix form by matrix earnings.

The player, knowing mixed strategy of nature ξ (θ) has the opportunity to determine average earnings nature reserve on it when deciding an action or another:

$$L(\xi, a) = M(L(\theta, a)) = \sum_{\theta} L(\theta, a) \cdot \xi(\theta)$$

Player most favorable action will be one that maximizes the gain medium, ie for which a0

This strategy will be called optimal strategy or Bayes strategy.

$$L\left(\xi, a_0\right) = \max L(\xi, a)$$

If exposed so far is reduced to a relatively simple statistical problem, namely, to find the best player strategy based on a priori the only information he has about the states of nature. In this case, the player can not specify the actual state of nature making experiences. Therefore, these games are called inexperienced.

Statistical unique experience games

A feature of the statistics game is, as noted, in the player's opportunity to broaden their knowledge by states of nature experiences. He must decide whether or not to be made experiences, their nature and their number.

Suppose that the player has decided to make a unique experience.

Z space results z_1, z_2, …, z_v of experience. z of Z obtained different results when nature is in state θ of Θ corresponds to a fixed probability p (z / θ) that satisfies the relationship:

$$\begin{cases} p(z/\theta) \geq 0, \text{ each z of Z} \\ \text{triple space Z consisting of experience results , space and states of nature } \Theta \\ \sum_{z} p(z/\theta) = 1 \end{cases}$$

conditional distribution p (z / θ) defined on Z for each θ in Θ is called the sampling area and note:

Φ = (*Z*, Θ, p)

It is convenient that the space Φ be given as a table of conditional probabilities p (z / θ).

The problems experienced player single decision will be based on the results z of Z of experience. It can thus develop a rule d of D that determines what a of A decision should be made on the result z of Z. This rule will be the function:

d: Z→A

which will be called the decision function. It assigns every result z_k in Z action a_j of A and then the earnings where nature is in state θ_i will be:

$L(\theta_i, a_j) = L(\theta_i, d(z_k))$

For given θ, z outcome of the experience will be a random variable with conditional probability p (z / θ). Then winnings L (θ, d) shall be conducted by the same probability p (z / θ) and will represent all random variables.

Estimating function decision d (z) for a given state of nature θ requires consideration of all possible outcomes of the experience, so it is necessary to determine the average earnings throughout the area results Z for each strategy d. The average earnings are called according risk and are:

For any state of nature θ of Θ and every d in D (where D is the space functions decision) will assign a value well defined ρ (θ, d), ie the function of risk will be

$$\rho(\theta, d) = M(L(\theta, d(z))) = \sum_{z \text{ in } Z} L(\theta, d(z)) \cdot p(z/\theta)$$

defined on the direct product Θ × D, as well as gain function L (θ, a) on Θ × A where games inexperienced. It comes therefore concludes that D plays in the game space with unique experience the same role as Area A in the game inexperienced.Similar methods to solve the two problems.

To determine optimal decision Player introduces the concept of winning hoped, representing average earnings in all states of nature θ of Θ:

$$\rho(\xi, d) = \sum_{\theta \text{ in } \Theta} \rho(\theta, d) \cdot \xi(\theta)$$

$$\rho\left(\xi, d_0\right) = \max_{d \text{ in } D} \rho(\xi, d)$$

Player optimal strategy called Bayes strategy will be

From the foregoing it follows that decisions D space has n^v elements.

Variant problem solving through statistical games with unique experience shown above is a variant (exemple ante) which uses a priori distribution of the states of nature.

Notes: Determining optimal strategies can be greatly simplified if recourse to (exeple post)option, ie to use a posteriori distributions of the states of nature.

$Z = \{z_1, z_2, \ldots, z_v\}$ Space experience results that does not give possibility to know the nature of his condition. This determination on the state of nature is that that experience is obtained in place of distribution a priori ξ (θ) a new distribution ξz (θ), which is called the probability distribution retrospectively defined the space Θ for a particular outcome z of Z.

According to a formula in the theory of probability, a posteriori probability can be determined as follows:

In this case we know the nature of state space Θ, A shares space regarded as pure player strategies and a probabilities of retrospective nature of strategies that take into account the results of z in Z of experience. When each action will match the gain medium, analog games inexperienced.

for each z in Z

$$L\left(\xi_z, a\right) = \sum_{\theta \text{ in } \Theta} L(\theta, a) \cdot \xi(\theta/z)$$

For fixed z choose the criterion Bayes optimal action (similar to games without experience):

$$L\left(\xi_z, a_0\right) = \max L(\xi, a)$$

Thus, choosing from z to Z the optimal action every day a_0 of A, form optimal strategy (the best decision) for variant (exemple post.)

Statistical sampling games

Unlike unique experience games, the best decision for statistical sampling games take the results of a given number N of consecutive experience. The player determines its actions after the completion of the string of samples (experiments).

But the player may act in another way: instead of conducting all N samples, each sample at the end he may decide to end the experiences and decisions of the multitude based on the information it already has or may decide to continue the experience. These games are called statistical sampling sequentially games. When

$$\xi_z(\theta) = \xi(\theta/z) = \frac{p(z/\theta) \cdot \xi(\theta)}{\sum_{\theta j \text{ in } \Theta} p(z/\theta j) \cdot \xi(\theta j)}$$

setting a threshold number of samples at the end of which decision of A should be mandatory when games are called games truncated statistical sampling sequence.

If statistical sampling gaming version is used only ex post. Such distributions are updated after each sample states of nature, to reflect a situation as close to reality.

The steps are similar and unique experience games, variant ex post.

Conclusions

Thus, employing the statistical and mathematical game theory and decision theory, it was determined the optimal strategy. The fact that the two versions of the mathematical instrument valuation and risk analysis provided the same results indicate soundness of the estimates and the quality problem situation analysis.

The analysis presented above could be complemented by other methods of evaluation, analysis and risk management and decision-making in situations of risk and uncertainty. For example, it might perform a sensitivity analysis optimal strategy in relation to the accuracy of initial data supplied (NPV for the three variants of the project, conditional probabilities of outcomes research marketing, distribution states of nature etc.). Such an analysis may be used as discrete data, in which case scenarios will work, as well as continuous data, in which case it will operate with one of the numerical modeling methods CNS (Computer Numerical Simulation). The results of this analysis indicate that the optimal strategy provided by using decision theory is stable, rigid relative to fluctuations initial data, or on the contrary, is very sensitive with respect to the original data. Additionally they can cause security indices of initial data supplied to the data producing optimal strategy change (critical data or critical points), established by the mathematical theory of decision.

Contribution of optimal strategy to gain maximum average

Below we present the decomposition of average earnings maximum structural components. Thus, the maximum average gain can be written as: $L(\xi_{z1},d_{optim}(z_1))*S_{z1}+ L(\xi_{z2},d_{optim}(z_2))*S_{z2}+ L(\xi_{z3},d_{optim}(z_3))*S_{z3}$,where $d_{optim} = d_{post}$. . I opted for writing d_{optim} expression by virtue results earlier demonstration of equivalence of the two solutions d_{post} and d_{ante} .In the future we will use the d_{optim} notation then adopt new strategy for optimum provided the mathematical game theory.

It appears that $S_{z1}+ S_{z2}+ S_{z3} = 1$ indeed

$$\sum_{z \text{ in } Z}\left(\sum_{\theta \text{ in } \Theta} p(z/\theta)\cdot\xi(\theta)\right)=\sum_{\theta \text{ in } \Theta}\left(\sum_{z \text{ in } Z} p(z/\theta)\cdot\xi(\theta)\right)=$$

$$=\sum_{\theta \text{ in } \Theta}\left[\left(\sum_{z \text{ in } Z} p(z/\theta)\right)\cdot\xi(\theta)\right]=\sum_{\theta \text{ in } \Theta}(1\cdot\xi(\theta))=1$$

So $L(\xi_{zi},d_{optim}(z_i))$ $(i=1..3)$ plays contribution and S_{zi} plays weights in calculating the maximum average gain (i = 1..3). Essentially, S_{zi} indicates the degree of influence (contribution) action optimal relative to each result of marketing research in the maximum average gain. The weight of each contribution to average earnings is the likelihood of the respective contribution (likelihood of z locked in all states of nature). As a result, the share will be calculated as the sum of the probabilities of occurrence of z for each state of nature. These results confirm the calculation of the share by S_{zi}

Earlier results can be generalized, by extension, for any number of research results z, θ states of nature, actions taken by the player *a*

The benefits of applying game theory and decision evaluation, analysis and risk management

- The assessment of the overall situation of the problem by taking into account a large number of influencing relevant factors
- Work with probability distributions continue, which increases precision, reliability and accuracy of information obtained and reduces the risk of taking wrong decisions because of the shortage, imprecision or inaccuracy of initial data
- Update distributions of probabilities is intended updating of historical data from real ones. Therefore accuracy of the estimates and the final results substantially increase credibility
- Combining both upstream and downstream to other methods of evaluation, analysis and risk management (e.g. Monte Carlo, sensitivity analysis, scenario analysis, graph theory, decision trees and relevance, numerical analysis technique BRANCH & BOUND etc.)
- Risk assessment taking wrong decisions because of the imprecision of the initial data via security indexes
- Establish materiality of the original data to ensure accurate decision making and to avoid taking wrong decisions because of the imprecision of the initial data
- combine both methods of evaluating and analyzing market-oriented (research and marketing studies), as well as processes oriented economic and financial aspects and technical (feasibility studies, technical analysis, economic and financial)
- Provide explicit criteria (express) decision making. For each problem situation provides optimal solution according to the results of marketing research on the economic situation and development prospects offered by this
- Provides a solution acceptable profitability dilemma – risk

Unlike game theory and decision, other methods of evaluation, analysis and risk management boils down to:

- Do not updated historical probability distributions at the current situation
- Not calculated materiality of the initial data that would guarantee the right decisions and avoid making wrong decisions
- Do not allow predicting the risk of taking wrong decisions because of the imprecision of the initial data
- Not specified critical values of baseline data and produce optimal strategy change

References:

Andrei T., Stancu S., Statistică — teorie şi aplicaţii, Editura All, Bucureşti, 1995
Florea I., Statistica, Universitatea Cluj,1986
Isaic–Maniu Al., Statistica, A.S.E., Bucureşti, 1984
Jaba Elisabeta, Jaba Octavian, *Posibilităţi de folosire a metodei statistice în analiza economică*, Revista de Statistică, nr. 4, 1982

Jaba Elisabeta, Statistica, Editura Economică, Bucureşti, 2000
Keller G., Warraek B., *Essentials of Business Statistics*, Wadsworth Inc., 1991