# STUDY ON THE EVOLUTION OF SOME FINANCIAL PRODUCTS BASED ON MARKOV CHAINS METHOD

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**Abstract:** *In probability theory it is known that Markov chain is frequently used in order to predict the future situations. Moreover, Markov chain theory is used to study the change rules of the economic phenomenons, to describe consumers' brand loyalty, in marketing for dynamic forecasts of market share, etc. In this study we introduced a Markov process model to track the evolution of some financial products for a commercial bank. At the same time we proposed a brief theory on related properties of Markov process based on which we continued the empirical research. Also, we used the mean first passage times, an another interesting and important long-term property of the Markov chain because they inform us about the steps that must be taken on average to move from one state to another.*

**Keywords:** Markov chains; transition probability matrix; the mean first passage times; financial products.

**JEL classification**: C22, G21

## 1. Introduction

Markov chain model has been widely studied and applied in order to predict the future situations. Moreover, Markov chain theory is used to study the change rules of the economic phenomenons, to describe consumers' brand loyalty (Uslu and Cam, 2000; Buciuman, 2011), in marketing for dynamic forecasts of market share (Armstrong and Farley, 1969; Dura, 2006; Zhang and Zhang, 2009; Lihong et al., 2014; Chan, 2015; Kovacs, 2015), etc.

In this study we introduced a Markov process model to track the evolution of some financial products for a commercial bank. Also, we proposed a brief theory on related properties of Markov process based on which we continued the empirical research, and we used the mean first passage times, an another interesting and important long-term property of the Markov chain.

## 2. Review of basic mathematical concepts

Markov chain is a special kind of stochastic process named after Russian mathematician, Andrei Andreevich Markov, where the outcome of an experiment depends only on the outcome of the previous experiment.

A sequence of random variables $\{X_t\} = X_1, X_2, \ldots$ is called Markov chain if the following equality holds:

$$P(X_{t+1} = j / X_1 = k_1, X_2 = k_2, \ldots, X_t = i) = P(X_{t+1} = j / X_t = i) \qquad \forall t = 0,1,\ldots \quad \text{and} \quad k_1, k_2, \ldots, i, j \in S,$$

where $S$ is named the state space of the chain. $S$ can be finite or countable. In other words this means that the future state $j$ at time $t+1$ only depends on state $i$ in which the system was in the previous time $t$, and this probability does not depend on the states in which it was before. We also can say that the Markov chain "memory" is very short: in every

moment it only "remembers" the previous period. This intuitive explanation is that the current state of the system includes any information relating to past states, which are needed to determine the following period of the state.
The following formulation, if the probability does not change with time

$$P(X_{t+1} = j / X_t = i) = P(X_1 = j / X_0 = i) \quad \forall t = 0,1,... \text{ and for all } i, j \in S$$

assumes that the Markov chain is stationary. This probability is known as an one-step transition probability, denoted by $p_{ij}$, and describes the probability of movement from state *i* to state *j*,

$$P(X_{t+1} = j / X_t = i) \overset{not}{=} p_{ij}$$

and has the following properties:

$$p_{ij} \in [0,1] \,, \ \sum_{j=1}^{n} p_{ij} = 1 \text{ for } i = 1,...,n \,.$$

The process can remain in the state it is in, and this occurs with probability $p_{ii}$.
The matrix notation facilitates modelling and subsequent calculations and it is considered to be a key component of a Markov chain.
An one-step transition probability matrix (say *P*) with *n* states can be written:

$$P = \begin{pmatrix} p_{11} & p_{12} & . & . & . & p_{1n} \\ p_{21} & p_{22} & . & . & . & p_{2n} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ p_{n1} & p_{n2} & . & . & . & p_{nn} \end{pmatrix}$$

Chapman-Kolmogorov equations allow the calculation of n-step transition probabilities by means of the matrix product; namely, to calculate the two-step transition probabilities, we generate the following product:

$$P^{(2)} = P \bullet P$$

The n-step transition probabilities can be determined based on the one-step transition probabilities:

$$P^{(n)} = P^n \quad \text{for} \quad n \geq 1$$

$$P^{(n)} = \{ p_{ij}^{(n)} \} \text{ where } p_{ij}^{(n)} = P(X_{t+n} = j / X_t = i)$$

and the *ij*th entry of the matrix $P^n$ gives the probability that the Markov chain, starting in state *i*, will be in state *j* after *n* steps.
We now consider the long-term behavior of a Markov chain when it starts in a state chosen by a probability distribution on the set of states, which we will call a probability vector, denoted by *v*. This probability vector with *r* components is a row vector whose entries are non-negative and the sum of the components is equal to 1.
If *v* is a probability vector which represents the initial state of a Markov chain, then we think of the *i*th component of *v* as representing the probability that the chain starts in state *i*. This is usually given as an initial probability vector.
It is easy to see that the probability that the chain is in state *i* after *n* steps is the *i*th entry in the vector

$$v^{(n)} = vP^n \,.$$

When $n$ is large enough, the transition probabilities stabilize, so that the probability that the system is found in a given state after many steps does not depend on the state in which it began. These probabilities are called stationary probabilities:

$$\pi_j = \lim_{n\to\infty} p_{ij}^{(n)}$$

The vector $\pi_j$ is a strictly positive probability vector (the components are all positive and their sum is equal to 1).

We can find the limiting vector $\pi_j$ (also called steady state vector) from the following equations system:

$$\pi_j = \sum_{i=1}^{r} \pi_i p_{ij} \quad , \quad j = 1,\dots,r$$

$$\sum_{j=1}^{r} \pi_j = 1$$

where $r$ is the total number of Markov chain states. If the equations are solved, we obtain unique solution. Stationary probabilities are an interesting characteristic of a system which can be modelled by a Markov chain because it allows us to know the percentage of time that the system is in each state.

Should be considered that the Markov chains model is a probabilistic technique, it does not provide a recommended decision.

The mean first passage times are another interesting long-term property in a Markov chain because they inform us about the steps that must be taken on average to move from one state to another.

Consider the mean first passage time from $i$ to $j$ and assume that $i \neq j$ , $i, j \in S$ . This may be computed as follows: take the expected number of steps required given the outcome of the first step, multiply by the probability that this outcome occurs, and then add them up. If the first step is to $j$, the expected number of steps required is 1; if it is to some other state $k$ ($k \in S \setminus \{j\}$), the expected number of steps required is $\mu_{kj}$ plus 1 for the step already taken. Thus, the $\mu_{ij}$ value can be determined as follows:

$$\mu_{ij} = p_{ij} + \sum_{k\neq j} p_{ik}(\mu_{kj} + 1)$$

and if we used that $p_{ij} + \sum_{k\neq j} p_{ik} = 1$, then

$$\mu_{ij} = 1 + \sum_{k\neq j} p_{ik}\mu_{kj} = 1 + \sum_{k\in S} p_{ik}\mu_{kj} - p_{ij}\mu_{jj} \ , \ \forall i, j \in S .$$

To calculate $\mu_{ij}$ we realise that it is necessary to know $\mu_{kj}$, so equations are added up until an equations system is formed which suffices to calculate the first passage time from $i$ to $j$, and some others. Usually these calculations are made with the help of computer programs. The theory presented above was adopted in the empirical study of this paper.

## 3. Research Methodology

Our research is an empirical one, based on the experience of some financial economic analysts of a commercial bank, considering that the data was provided by them. In this study we used Matlab for calculations, which is an interactive program for numerical computation and data visualization. Also, for calculations, was used Microsoft Excel.

## 4. The empirical research

Further on we tried to model the annual evolution of a set of 10 financial products by means of a Markov chain. The 10 financial products which the bank offers to individuals and companies are: personal loan, car loan, mortgage / real estate loan, refinancing credit, deposits, credit card, debit card, business card, Gold card, current account and attached services. We have classified these financial products into three categories:

- 1 product is considered appealing: highly profitable in the short term, but uncertain in the mid and long terms.
- 7 products are considered secure: mean estimated profitability for the short, mid and long terms.
- 2 products are considered insecure: their profitability varies, even in the short term.

We estimates that an appealing product still has the same probability of remaining appealing the next year or becomes a secure or an insecure product. Conversely, the secure product becomes an appealing one with a 0.25 probability, while its probability of remaining secure is 0.5. Insecure products have a 0.1 probability of being considered secure products and of 0.8 of still being considered insecure products.
Knowing the dates we can establish the initial transition probability matrix. The state space of the chain is $S = \{S_1, S_2, S_3\}$, where $S_1$ represents appealing products, $S_2$ represents secure products, $S_3$ represents insecure products.

**Table 1:** The transition probability matrix

| | $S_1$ | $S_2$ | $S_3$ |
|---|---|---|---|
| $S_1$ | 1/3 | 1/3 | 1/3 |
| $S_2$ | 0.25 | 0.5 | 0.25 |
| $S_3$ | 0.1 | 0.1 | 0.8 |

First we want to find the mean recurrence times for each state of the Markov chain.
Using Matlab, the expected first passage times are:

**Table 2:** The expected first passage time

| | $S_1$ | $S_2$ | $S_3$ |
|---|---|---|---|
| $S_1$ | 5.67 | 5.33 | 3.33 |
| $S_2$ | 6 | 4.25 | 3.67 |
| $S_3$ | 8 | 7.67 | 1.7 |

Source: author's calculation

After calculations, we find that a mean of 5.67 years is expected for an appealing product to be once again considered appealing, 4.25 years is needed for a secure product to be once again considered secure, and a time of 1.7 years is required for an insecure product to be once again considered insecure. The average time that must pass for an appealing product to become a secure product is 5.33 years, and to become an insecure product is 3.33 years.
As the Markov chain is finite and ergodic, it can be concluded that a stationary distribution exists. To calculate it, the following equations system is considered:

$\pi_1 = \frac{1}{3}\pi_1 + \frac{1}{3}\pi_2 + \frac{1}{3}\pi_3$

$\pi_2 = 0.25\pi_1 + 0.5\pi_2 + 0.25\pi_3$

$\pi_3 = 0.1\pi_1 + 0.1\pi_2 + 0.8\pi_3$

$\pi_1 + \pi_2 + \pi_3 = 1$

By solving the linear equations system, the following stationary transition probabilities are obtained:

$\pi_1 = 0.1765$

$\pi_2 = 0.2353$

$\pi_3 = 0.5882$

The probability of an appealing product in the long term is 17.65%, and for a secure product it is 23.53%. If we want to know how many products will be in the long term, by considering the initial distribution, then we found the following:

$\pi_1 = 0.1765 \rightarrow 10 \bullet \pi_1 = 1.765$

$\pi_2 = 0.2353 \rightarrow 10 \bullet \pi_2 = 2.353$

$\pi_3 = 0.5882 \rightarrow 10 \bullet \pi_3 = 5.882$

In the long term, there are less secure products, but on the other hand there are more insecure ones. The number of appealing products almost remained unchanged.
Next, we consider the two-step transition probability matrix.

**Table 3:** The two-step transition probability matrix

| | **$S_1$** | **$S_2$** | **$S_3$** |
|---|---|---|---|
| **$S_1$** | 0.2278 | 0.3111 | 0.4611 |
| **$S_2$** | 0.2333 | 0.3583 | 0.4083 |
| **$S_3$** | 0.1383 | 0.1633 | 0.6983 |

Source: author's calculation

From the two-step transition probability matrix we can observe the probabilities after 2 years. The probability that an appealing product continues to be appealing after 2 years is 22.78%. After 2 years the probability that an appealing product, respectively a secure product becomes an insecure product show an increasing tendency, 46.11%, respectively 40.83%. After 2 years the probability that an appealing product becomes a secure product show a decreasing tendency, 31.11%, respectively an insecure product becomes a secure product show an increasing tendency, 16.33%.
We present below the evolution of the three financial products for 14 years.

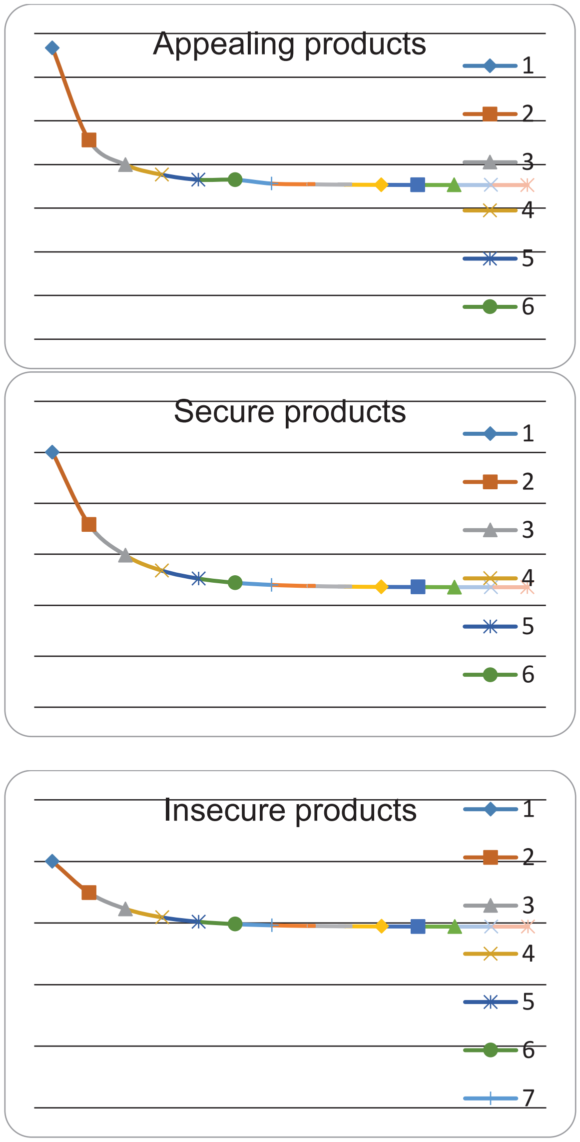

**Figure 1:** The evolution of the three financial products (for 14 years)
Source: made by the author

The probability of an appealing product in the long term decreases from 33.33% to 17.65%, and for a secure product it decreases from 50% to 23.53%, and for an insecure product it decreases from 80% to 58.82%.

**5. Conclusions**

In this study we showed that the Markov chain model can be used to model the annual evolution of some financial products. We answered a few important questions by finding the expected first passage times, the stationary transition probabilities, the two-step transition probability matrix.

The process what we modeled in this study was stationary. In case of stationary time series the use of Markov chain is simpler, furthermore, is more accurate and efficient as well. We also supposed that the transition probabilities do not change over time and the number of states is stable over time.

Markov chain model provides probabilistic information about a decision situation that can aid the decision maker in making a decision.

**References**

Armstrong, J. S., Farley, J. U. (1969) A note on the use of Markov chains in forecasting store choice, *Management Science*, 16(4), pp. B-281-B-285.

Bucea-Manea-Ţoniş, R., Bucea-Manea-Ţoniş, R. (2013) Computer assisted simulation of market shares for competing products using Markov chains, *Buletinul AGIR*, nr. 3 (iulie-septembrie), pp. 109-114.

Buciuman, C. F. (2011) Uses of Markov Chains in Forecasting Brand Loyalty, *Buletinul Ştiinţific al Universităţii "Politehnica" din Timişoara, Seria Management. Inginerie economică. Ingineria Transporturilor*, Tom 56(70), Fascicola 1.

Chan, K. C. (2015) Market share modelling and forecasting using Markov chains and alternative models, *International Journal of Innovative Computing, Information and Control*, 11(4), pp. 1205-1218.

Dura, C., (2006), The use of Markov Chains in Marketing Forecasting, *Annals of the University of Petrosani, Economics*, 6, pp. 69-76.

Kovacs (Kiss), M. (2015) Study on using Markov chains in brand forecasting, Conferinţa Internaţională: Austeritate sau dezvoltare? Lecţii din criza economică globală – AUS2015, 30-31 octombrie, Bucureşti.

Lihong, L.,Jie, S., Yan, L., Hai, X. (2014) Mathematical model based on the product sales market forecast of markov forecasting and application, *Journal of Chemical and Pharmaceutical Research*, 6(6), pp. 1359-1365.

Uslu, A., Cam, T. (2000) *Analysis of Brand Loyalty with Markov Chains,* Challenges for Business Administrations in the New Millenium, First International Joint Symposium on Business Administration, Gökceada, Canakkale-Turkey 1-3 June, pp. 583-591.

Zhang, D., Zhang, X. (2009) Study on forecasting the stock Market Trend Based on Stochastic Analysis Method, *International Journal of Business and Management*, 4(6), pp. 163-170.