PRICING AND ASSESSING UNIT-LINKED INSURANCE CONTRACTS WITH INVESTMENT GUARANTEES

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Abstract: *One of the most interesting life insurance products to have emerged in recent years in the Romanian insurance market has been the unit-linked contract. Unit-linked insurance products are life insurance policies with investment component. A unit-linked life insurance has two important components: protection and investment. The protection component refers to the insured sum in case of the occurrence of insured risks and the investment component refers to the policyholders' account that represents the present value of the units from the chosen investment funds. Due to the financial instability caused by the Global Crisis and the amplification of market competitiveness, insurers from international markets have started to incorporate guarantees in unit-linked products. So a unit- linked life insurance policy with an asset value guarantee is an insurance policy whose benefit payable on death or at maturity consists of the greater of some guaranteed amount and the value of the units from the investment funds. One of the most challenging issues concerns the pricing of minimum death benefit and maturity benefit guarantees and the establishing of proper reserves for these guarantees. Insurers granting guarantees of this type must estimate the cost and include the cost in the premium. An important component of the activity carried out by the insurance companies is the investment of the premiums paid by policyholders in various types of assets, in order to obtain higher yields than those guaranteed by the insurance contracts, while providing the necessary liquidity for the payment of insurance claims in case of occurrence of the assumed risks. So the guaranteed benefits can be broadly matched or immunized with various types of financial assets, especially with fixed-interest instruments. According to Romanian legislation which regulates the unit-linked life insurance market, unit-linked life insurance contracts pass most of the investment risk to the policyholder and involve no investment risk for the insurer. Although the Romanian legislation authorizes the Romanian insurers to offer unit-linked contracts without investment guarantees, this research provides a proposal of a theoretical and empirical basis for pricing the unit-linked insurance contracts with incorporated investment guarantees.*

Keywords: Unit-linked products; Investment guarantees; Black-Scholes Model; Call options; Treasury bills.

JEL classification: G22, G17, G12, C58, C87.

1. Introduction

Pricing of unit-linked life insurance contracts has generated much interest among researchers and practitioners in the last two decades (Romanyuk, 2006). Unit-linked contracts have two important components: protection and investment. The protection component refers to the insured sum in case of the occurrence of insured risks and the investment component refers to the policyholder's account that represents the present value of the units from the chosen investment funds (Gavriletea, 2009). So the payoff in such contracts contains both financial and insurance risk elements, which have to be priced so that the resulting premium is fair to both the seller (insurer) and the buyer (policyholder) of the contract (Romanyuk, 2006). Due to the financial instability caused by the Global Crisis and the amplification of market competitiveness, insurers from international markets have started to incorporate guarantees in unit-linked products. Investment guarantees are very popular features in life insurance policies because in addition to paying a death benefit or a maturity benefit, these policies are tied to the return of an underlying asset or an actively managed portfolio. Thus, the policy also acts as an investment because the investor's capital is credited with a minimum return. In exchange for this protection, the policyholder pays a higher premium, reflecting the market risk assumed by the insurance company (Augustyniak and Boudreault, 2012). An important component of the activity carried out by the insurance companies is the investment of the premiums paid by policyholders in various types of assets, in order to obtain higher yields than those guaranteed by the insurance contracts, while providing the necessary liquidity for the payment of insurance claims in case of occurrence of the assumed risks. So the guaranteed benefits can be broadly matched or immunized with various types of financial assets, especially with fixed-interest instruments (Hardy, 2003).

The objective of this research: although the Romanian legislation authorizes the insurance companies to offer unit-linked contracts without investment guarantees, this research provides a proposal of a theoretical and empirical basis for pricing and assessing unit-linked insurance contracts with investment guarantees. This study contributes to the existing literature regarding the issue of appropriate pricing of life insurance contracts and hedging of the risks involved, with an exclusive focus on the unit-linked life insurance contracts with investment guarantees.

The structure of this paper is as follows: Section 2 discusses some previous research on the issue. Section 3 describes the main categories of investment guarantees commonly used in unit-linked insurance and presents some of the contracts that offer investment guarantees. Section 4 outlines the methodology. The sample data and the empirical results are presented in Section 5. Section 6 provides a summary of the main findings and some concluding remarks.

2. Literature review

There is an extensive literature on the pricing, hedging and risk management of these contracts. See for example, Nielsen & Sandmann (1995), Tiong (2001), Grosen (2002), Hardy (2003), Argesanu (2004), Biffis & Millossovich (2006), Gaillardetz (2006), Romanyuk (2006), Boyle and Tian (2009), Reichenstein (2009), Keneley (2010), Mahayni and Schlogl (2010), Gatzert et al.(2011), etc.

Brennan and Schwartz (1976), Boyle and Schwartz (1977), and Brennan and Schwartz (1979) were the first articles that elegantly described some of the option elements of life insurance products and demonstrated how the relatively young option pricing theory of Black and Scholes (1973) could be applied to value these contracts. Brennan and Schwartz (1976), Brennan and Schwartz (1979) consider an equity-linked life insurance policy with an asset value guarantee and determine the value of such policy using the economic concept of equilibrium pricing. Boyle and Schwartz (1977) work out a similar solution for death benefit and maturity benefit guarantee contracts, which pay the larger of a fixed guarantee or value of some risky fund at expiration of the contract or upon the death of the policyholder. Delbaen (1986) extends previous articles by proposing Monte-Carlo simulation to price fixed term equity-linked contracts with guarantee, for which premiums have to be paid periodically and survival probability of the client is factored into the value of the contract.

Bacinello and Ortu (1993) further build on the above papers by considering the case of equity-linked contracts where guarantees are determined endogenously based on the premiums paid, as opposed to being specified exogenously, as in Brennan and Schwartz (1976), Brennan and Schwartz (1979) and Delbaen (1986). Ekern and Persson (1996) calculate premiums for a large variety of equity-linked contracts, including those with payoffs where the contract owner chooses the larger of the values of two risky assets (and possibly a guaranteed amount) at maturity of the contract. Boyle and Hardy (1997) examine the pricing of and reserving for maturity guarantees for policies where the policyholders' premiums are invested in a specified portfolio which is guaranteed not to fall below a certain level at maturity. Tiong (2000) examines the pricing, hedging and accounting of equity-indexed annuities (EIAs). Moeller (2001) examines a portfolio of equity-linked life insurance contracts and determines risk-minimizing strategies in a discrete-time setting for the Cox-Ross-Rubinstein model.

Hardy (2003) discusses the modelling and risk management for equity-linked life insurance; the focus of his research is on stochastic modeling of embedded guarantees that depend on equity performance. Argesanu (2004) focuses on the risk analysis and hedging of variable annuities in incomplete markets. Romanyuk (2006) describes the problem of appropriate pricing of equity-linked life insurance contracts and hedging of the risks involved, and proposes the use of two types of imperfect hedging techniques: quantile and efficient hedging. Gaillardetz (2006) introduces a pricing method for equity-indexed annuities and valuates these products by pricing its death benefits and survival benefits separately. Boyle and Tian (2008) examine the optimal design of equity-linked products with probabilistic constraint. Mao and Ostaszewski (2008) examine the pricing of equity-linked life insurance including a minimum interest rate guarantee in a partial equilibrium framework.

Dickson, Hardy and Waters (2009) have developed the theory to measure and manage risks that are contingent on demographic experience as well as on financial variables. Quittard-Pinon and Randrianarivony (2009) focus on the pricing of a particular equity-linked life insurance contract where the conditional payoff to the policyholder is the maximum of two risky assets. Bernard and Boyle (2011) propose the establishment of a natural hedge to reduce the volatility risk of an insurance company's liabilities. Sweet (2013) focuses on the pricing and evaluating of the equity-linked annuities.

3. Types of contracts that offer investment guarantees

In this section the authors present the various types of investment guarantees commonly used in unit-linked insurance and describe some of the contracts that offer investment guarantees as part of the benefit package. The unit-linked contracts offer some element of participation in an underlying index or fund or combination of funds, in conjunction with one or more guarantees. Without a guarantee, unit-linked insurance involves no risk to the insurer, which merely acts as a steward of the policyholders' funds. These fixed-sum risks generally fall into one of the following major categories:

The guaranteed minimum maturity benefit (GMMB): guarantees the policyholder a specific monetary amount at the maturity of the contract. This guarantee provides downside protection for the policyholder's funds, with the upside being participation in the underlying stock index. A simple GMMB might be a guaranteed return of premium if the stock index falls over the term of the insurance (with an upside return of some proportion of the increase in the index if the index rises over the contract term). The guarantee may be fixed or subject to regular or equity-dependent increases.

The guaranteed minimum death benefit (GMDB): guarantees the policyholder a specific monetary sum upon death during the term of the contract. Again, the death benefit may simply be the original premium, or may increase at a fixed rate of interest. More complicated or generous death benefit formulae are popular ways of tweaking a policy benefit at relatively low cost.

With ***the guaranteed minimum accumulation benefit (GMAB)*,** the policyholder has the option to renew the contract at the end of the original term, at a new guarantee level appropriate to the maturity value of the maturing contract. It is a form of guaranteed lapse and re-entry option.

The guaranteed minimum surrender benefit (GMSB) is a variation of the guaranteed minimum maturity benefit. Beyond some fixed date the cash value of the contract, payable on surrender, is guaranteed. A common guaranteed surrender benefit in Canadian segregated fund contracts is a return of the premium.

The guaranteed minimum income benefit (GMIB) ensures that the lump sum accumulated under a separate account contract may be converted to an annuity at a guaranteed rate. When the GMIB is connected with an equity-linked separate account, it has derivative features of both equities and bonds. In the United Kingdom, the guaranteed-annuity option is a form of GMIB. A GMIB is also commonly associated with variable-annuity contracts in the United States.

This section also describes the main types of insurance contracts that offer investment guarantees.

Segregated fund contracts: the segregated fund contract in Canada has proved an extremely popular alternative to mutual fund investment. The basic segregated fund contract is a single premium policy, under which most of the premium is invested in one or more mutual funds on the policyholder's behalf. The contracts all offer a GMMB and a GMDB of at least 75 percent of the premium and 100 percent of premium is common. The name "segregated fund" refers to the fact that the premium, after deductions, is invested in a fund separate from the insurer's funds. The insurer usually offers a range of funds, including fixed interest, balanced (a mixture of fixed interest and equity), broad-based equity, and perhaps a higher-risk or specialized equity fund.

The variable-annuity (VA) contract is a separate account insurance, very similar to the Canadian segregated fund contract. Premiums net of any deductions are invested in subaccounts similar to the mutual funds offered under the segregated fund contracts. GMDBs are a standard contract feature; GMMBs were not standard a few years ago, but are beginning to become so. They are known as VAGLBs or variable-annuity guaranteed living benefits. Death benefit guarantees may be increased periodically.

Unit-linked insurance resembles segregated funds, with the premium less deductions invested in a separate fund. Some unit-linked contracts associated with pensions policies carry a guaranteed annuity option, under which the fund at maturity may be converted to a life annuity at a guaranteed rate.
The equity-indexed annuity (EIA) offers participation at some specified rate in an underlying index. The contract will offer a guaranteed minimum payment of the original premium accumulated at a fixed rate; a rate of 3 percent per year is common. EIAs are generally rather shorter in term than unit-linked products, with seven-year policies being typical.
Equity-linked insurance: these contracts resemble the EIAs, with a guaranteed minimum interest rate applied to the premiums, along with a percentage participation in a specified index performance (Hardy, 2003).

4. Methodology

Soon after the celebrated papers by Black and Scholes (1973) and Merton (1973) on the pricing of call options, the topic of pricing of unit-linked insurance contracts became popular (Romanyuk, 2006). One of the most intriguing issues concerns the pricing of minimum death benefit or maturity benefit guarantees and the establishing of proper reserves for these guarantees.
The essence of mortality risk is that individual lives are independent so that risks may be nearly eliminated by pooling or diversification. Investment risk on the other hand affects all contracts simultaneously and in the same direction. A single event, a dramatic decline in stock prices simultaneously will render the insurer liable under the maturity guarantee of all its maturing contracts. Insurers granting guarantees of this type must estimate the cost and include this cost in the premium (Boyle and Schwartz, 1977).
Insurance companies should purchase bonds (instruments with fixed income) and call options to cover the minimum guarantee and to meet the market return portion of the payoff (Tiong, 2000).
An option is a security which gives its owner the right to buy (or sell) an asset at a fixed price, under certain conditions, within a fixed period of time (Boyle and Schwartz, 1977). The holder of a European call option on a stock has the right (but not the obligation) to buy an agreed quantity of that stock at a fixed price, known as the strike price, at a fixed date, known as the expiry or maturity date of the contract (Dickson, Hardy and Waters, 2009).
Let S_t denote the price of the stock at time t. The holder of a European call option on this stock with strike price K and maturity date T would exercise the option only if $S_t > K$, in which case the option is worth $S_t - K$ to the option holder at the maturity date.

The option would not be exercised at the maturity date in the case when $S_t < K$, since the stock could then be bought for a lower price in the market at that time. Thus, the payoff at time T under the option is:

$$(ST - K) = \max(ST - K, 0) \quad (1)$$

For a call option, at-the-money means that the stock and strike prices are roughly equal (Hardy, 2003).

Black and Scholes(1971) show that it is possible to form a portfolio of an option and a share so that the rate of return on the combined portfolio is non-stochastic or riskless (Boyle and Schwartz, 1977). In the Black-Merton-Scholes (BMS) framework, one can replicate the payoff of an investment guarantee, which is a essentially a put option, by trading in the underlying stock and the risk-free asset (Augustyniak and Boudreault, 2012).

The Black-Scholes-Merton framework for option valuation is a continuous time model, and is based on more sophisticated market assumptions: the market model should follow a geometric Brownian motion (GBM), there should be no market frictions (no transaction costs and no constraints on trading) (Augustyniak and Boudreault, 2012), short selling is allowed without restriction, and borrowing and lending rates of interest are the same, trading is continuous and interest rates are constant (Hardy, 2003). If we consider a single premium unit-linked contract providing a guaranteed amount g on death or maturity; g may be a function of time or it may be a constant. So when a claim arises the amount payable is the greater of g and the market value of the units purchased in the reference portfolio. Also the date on which the contract will become a claim is known with certainty. This situation corresponds precisely to that of a combination of a call option on a non-dividend paying stock plus the payment of the fixed amount g. If the value of the reference portfolio is denoted by x, the benefit can be expressed as follows:

$$Benefit = \max(g, x) = g + \max(x - g, 0) \quad (2)$$

5. Empirical results

In this section the authors have performed an empirical simulation regarding the pricing and assessing of the unit-linked insurance policy with investment guarantee.

A life insurance company is issuing single-premium unit-linked insurance policies with investment guarantees with the following characteristics:

Table 2: The characteristics of unit-linked insurance contracts with investment guarantees issued by a life insurance company

Unit-linked Single Premium for 1 Insurance Policy (LEI)	10,000
Number of contracts	100
Unit-linked Single Premium for the Entire Portfolio of Insurance Policies (LEI)	1,000,000
Insurance sum in the event of a death / contract (LEI)	25,000
The Age of the Insured Person (years)	35-45
Guaranteed Minimum Crediting Rate (%)	3.00
Unit-linked Maturity (years)	10
Current level of BET Index	6,359
Price of At-The-Money BET Index Call Option with 10 Year Maturity (LEI)	3,643
Current Rate on 10 Year, Treasury Bills (%)	6.99

Source: Authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

When the insurance company receives the premiums from the policyholders (a total amount of 1,000,000 LEI), it will invest sufficient financial funds in a Treasury bills portfolio in order to make sure that it can guarantee the minimum return (3.00%) to the policyholders.

The authors have performed Excel DerivaGem test, 2.01 version, to calculate the call option price. The call option price is calculated using the Black & Scholes formula:

$$C = N(d_1) \cdot S - N(d_2) \cdot Ke^{-rT} \quad (3)$$

$$d_1 = \frac{ln\left(\frac{S}{K}\right) + rt + \frac{\sigma^2 t}{2}}{\sigma\sqrt{t}} \quad (4)$$

$$d_2 = d_1 - \sigma\sqrt{t} \quad (5)$$

Where:

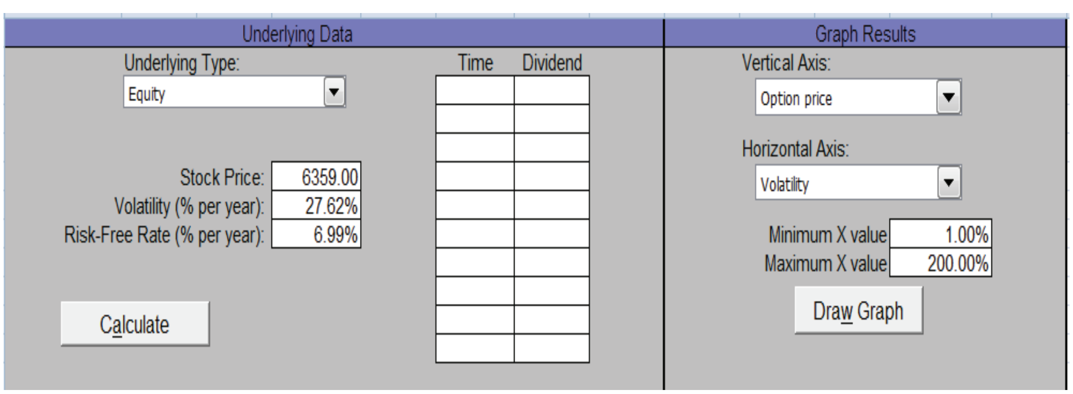
- N is the cumulative distribution function for the standard normal distribution
- S is the spot price of the underlying stock
- K is the strike price
- r is the risk-free rate of return
- t is the time to maturity
- σ is the volatility of the returns of the underlying stock.

The data of the study consists of daily closing values of BET index. BET is the reference index for the Bucharest Stock Exchange (BVB) market. BET is a free float weighted capitalization index of the most liquid 10 companies listed on the BVB regulated market. The index methodology allows BET to be a good underlying for derivatives and structured products.

All the closing prices of Bet index are collected from BVB database. The daily data is taken from April 1, 2004 to April 1, 2014. We have calculated the daily standard deviation of returns series using software Eviews: $\sigma_{daily} = 1.74\%$ and then we have obtained the annual volatility for the Romanian stock market, using the following formula: if the daily logarithmic returns of a stock have a standard deviation of σ_{daily} and the time period of returns is *N*, the annual volatility is:

$$\sigma_{annual} = \frac{\sigma_{daily}}{\sqrt{N}} \quad (6)$$

A common assumption is that *N* ≅ 1/252 (there are 252 trading days in any given year). The annual volatility is 27.62%.



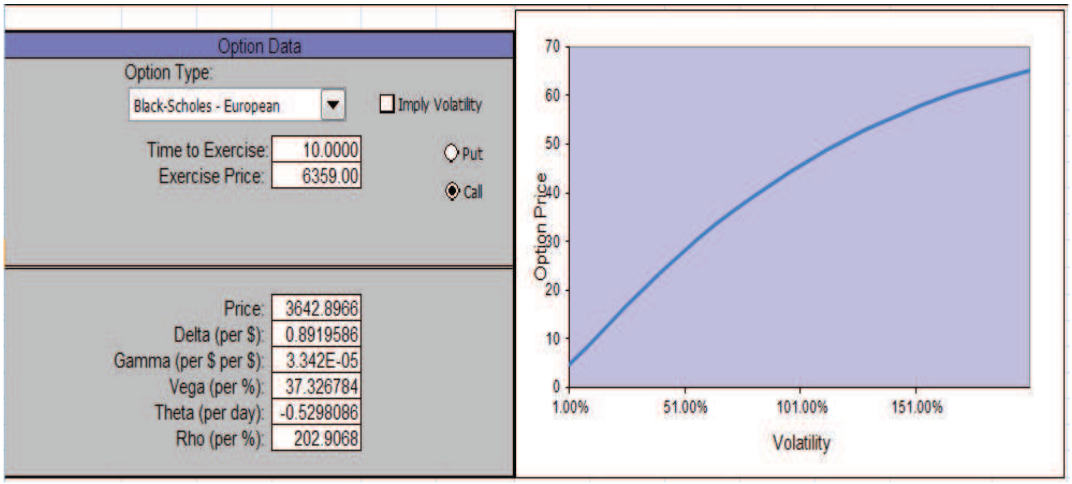


Figure 10: The calculation of the call option price
Source: Authors' processing; this figure is imported from Excel DerivaGem 2.01.

Therefore the price of a call option is Call (T=10, K=6,359, S=6,359) = 3,643 LEI.
The insurance company has to fulfill the following steps:

- How much should the insurance company invest in Treasury bills today to be able to meet the guaranteed minimum rate?

In 10 years the insurance company is required to obtain the following amount of money to guarantee the minimum return to the policyholders: 1,343,916.38 LEI.

$$1{,}343{,}916.38 = 1{,}000{,}000(1 + 0.03)^{10} \qquad (7)$$

So today the insurance company should invest the following amount of money in Treasury bills in order to be able to provide the guaranteed minimum rate (3.00%): 683,817.75 LEI.

$$683{,}817.75 = 1{,}343{,}916.38 : (1 + 0.0699)^{10} \qquad (8)$$

- How many BET Index Call Options should the company purchase today?

The insurance company can invest 316,182.25 LEI (1,000,000-683,817.75) in call options.
So today the insurer should purchase 87 call options at the price of 3,643 LEI.

Table 3: The investment in Unit-linked products in 10 years

Scenario	**Pessimistic**	→	**Optimistic**
BET Index Value in 10 Years	3,174	6,359	12,718
Ending Value of Treasury Bills Portfolio (LEI)	1,343,916.38	1,343,916.38	1,343,916.38
Ending Value of Index Call Option Portfolio (LEI)	0	0	553,233
Ending Value of Unit-Linked Products (LEI)	1,343,916.38	1,343,916.38	1,897,149.38
Annualized Return of Unit-Linked (%)	3.00%	3.00%	6.61%
Annualized Return of BET (%)	-6.71%	0	7.18%

Source: Authors' processing

As we can see in the table above, in the case of an optimistic scenario, the ending value of unit-linked products will be 1,897,149.38 LEI and the annualized return of unit-linked products is 6.61% (which is above the guaranteed minim return of 3.00%).

6. Conclusions

One of the most interesting life insurance products to have emerged in recent years in the Romanian insurance market has been the unit-linked contract (Boyle and Schwartz, 1977). Unit-linked insurance with guarantee provides downside protection with a guaranteed minimum return (Josephy, Kimball and Steblovskaya, 2011). So a unit-linked life insurance policy with an asset value guarantee is an insurance policy whose benefit payable on death or at maturity consists of the greater of some guaranteed amount and the value of the units from the investment funds (Brennan and Schwartz, 1979).

A unit-linked contract is wrapped with some guarantees such as: the guaranteed minimum maturity benefit (GMMB), the guaranteed minimum death benefit (GMDB), the guaranteed minimum accumulation benefit (GMAB), the guaranteed minimum income benefit (GMIB) and the guaranteed minimum surrender benefit (GMSB) (Gaillardetz, 2006).

According to Romanian legislation which regulates the unit-linked life insurance market, unit-linked life insurance contracts pass most of the investment risk to the policyholder and involve no investment risk for the insurer. Although the Romanian legislation authorizes the Romanian insurers to offer unit-linked contracts without investment guarantees, this research provides a proposal of a theoretical and empirical basis for pricing the unit-linked insurance contracts with incorporated investment guarantees.

The Black-Scholes Model (1971) shows that it is possible to form a portfolio of an option and a share so that the rate of return on the combined portfolio is non-stochastic or riskless (Boyle and Schwartz, 1977). The guaranteed benefits can be broadly matched or immunized with various types of financial assets, especially with fixed-interest instruments (Hardy, 2003). In the situation corresponding precisely to a combination of a call option on a non-dividend paying stock plus the payment of the fixed guaranteed amount, the benefit payable can be expressed as the greater of the guaranteed amount and the market value of the units purchased in the reference portfolio (Boyle and Schwartz, 1977).

According to the empirical results, in the case of an optimistic scenario, the ending value of unit-linked products will be 1,897,149.38 LEI and the annualized return of unit-linked products is 6.61% (which is above the guaranteed minim return of 3.00%). On the other hand in the case of a pessimistic scenario, the ending value of unit-linked products will be 1,343,916.38 LEI and the annualized return of unit-linked products is 3.00% (which is the guaranteed minim return). Besides the gains provided by the investment, the insured person can benefit also from the protection component (the insured sum is 25,000 LEI).

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