# THE ROLE OF VALUE AT RISK IN THE MANAGEMENT OF ASSET AND LIABILITIES

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*ALM is the management of risk at enterprise level, the models used in ALM can be static or dynamic: single period-static models, multiple period static model, single period stochastic model, multi period stochastic model. While single period-static don't incorporate the dynamic of the economical changes the multiple period-static models are an extension of the single period-static model, the most common used are multi-period stochastic which model the evolution of financial series in time and the assets and liabilities using different types of probability distributions (Student, GED). Highly correlated with ALM is the Value at Risk which can be used as and function to be minimized in ALM models. In the Value at Risk methodology the estimation models are classified as: parametric, nonparametric, semi-parametric; we present the parametric models (GARCH models) used in Value at Risk and the connections that can be established between ALM models and Value at Risk. We present the Conditional Value-at-risk and offer and example on how to calculate CVaR .*
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## 1.Introduction

Asset liabilities management (ALM) is defined by (Rosen & Zenios, 2006) as the strategic management of the balance sheet, (Uyemura, Van Deventer, & Foundation, 1993) define ALM as the management of income and expenses with respect to maximizing earnings, adjusted to risk factors, given the long term interest of the shareholders, also ALM manages the risk due to mismatches between assets and liabilities. The main purpose of ALM is the management of risk, in the context of enterprise risk management, having different objectives : profitability, liquidity, solvency, Conditional Value-at-risk (Ferstl & Weissensteiner, 2011)□. The first steps in ALM where made by (Markowitz, 1952) who treats the management of assets in an efficient portfolio, while (Sharpe & Tint, 1990) discuss the correlation and the associated risk between assets and liabilities. The first ALM models where developed from the early '80s (Uyemura et al., 1993). For (Kusy & Ziemba, 1986) ALM represents a cost/profit analysis between the risk level , earnings and liquidity; using a stochastic linear programming model they concluded these models are more robust and efficient then the previous linear programming models. (Giokas & others, 1991) test a linear programming model on the Commercial Bank of Greece which takes into consideration the institutional and legal framework, the financial position, showing that using ALM the risk level can be reduce in banks.

The ALM models can be classified (Zenios & Ziemba, 2007), in terms of the period and the random variables, as : single period-static models, multiple period static model, single period stochastic model, multi period stochastic model.

Single period-static models, which started with balance sheet immunization (Redington, 1952), and are better know as GAP analysis have some drawbacks because these models don't incorporate the dynamic of the economical changes. Multiple period-static models are an extension of the single period-static model, in which the period analyzed in more than one year.

Single period stochastic models include the analysis of risk-return (Markowitz, 1952) which treats the assets allocation. While (Sharpe & Tint, 1990) also analyze the liabilities fluctuations

and their relation with asset allocation. Both (Markowitz, 1952; Sharpe & Tint, 1990) use normal law in order to represents the assets and liabilities, the non-normality of financial series is presented by (Zenios, 1995) ,using asymmetric and non-normal distribution for the financial series he finds that this models outperform.
Multiperiod stochastic models are the most common in ALM, for e.g. the stochastic programming (Carino et al., 1994) the Russel Yasuda Carino (Dempster & Consigli, 1996) and Promoeteia model (Consiglio, Flavio, Cocco, Zenios, & others, 2007), they model the evolution of financial series in time and the assets and liabilities are considered as having different types of probability distributions (Student, GED).
(Ferstl & Weissensteiner, 2011) apply a multi-period stochastic linear programming model with re-allocation, the function to be minimized is the CVaR of shareholder value, which is the difference between mark to market value of assets and the present value of liabilities.

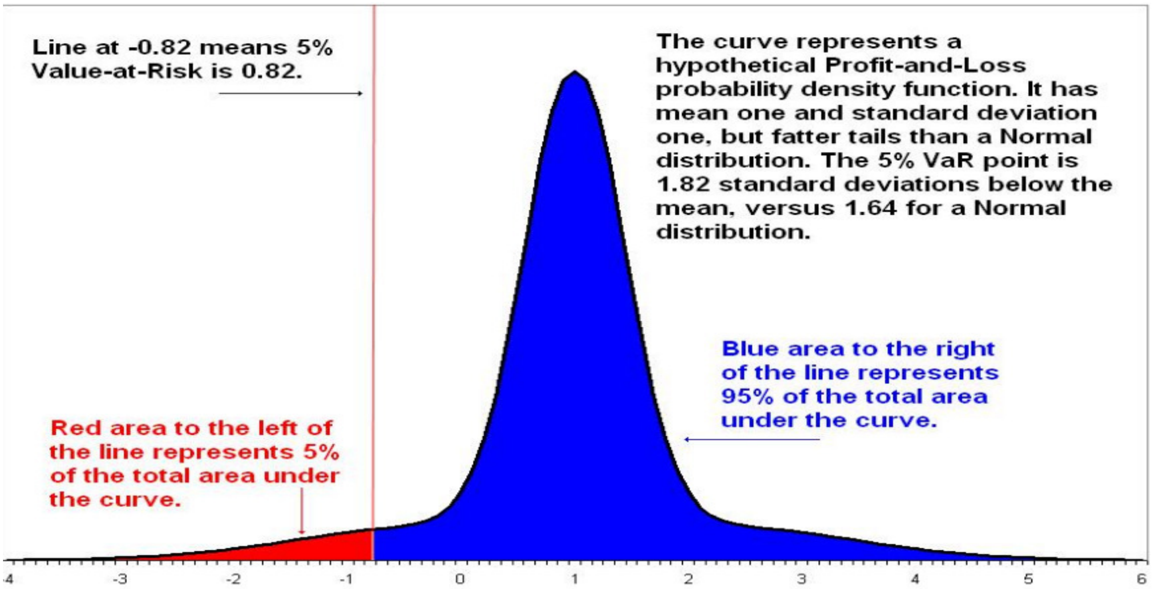
## 2.Value at Risk

The Value-at-risk is defined by (McNeil et all, 2005) as at "... some confidence level $\alpha \in (0,1)$ the VaR of the portfolio at the confidence level $\alpha$ is given by the smallest number l such that the probability that the loss L exceeds l is not larger than $(1-\alpha)$ ." Or mathematically we can write VaR as probability:

$$P_{VaR} = P(l < -VaR) = \int_{-\infty}^{-VaR} P_T(l)\,dl \quad (1)$$

In Figure 1 we have a hypothetical distribution functions of Profit and Loss (P/L) where P/L has a leptokurtic distribution with mean 1 and standard deviation 1, then the Value-at-risk represent the percentile (in red) of losses.

Fig.1. Value-at-risk graphic representation




Source: http://en.wikipedia.org/wiki/Value_at_risk

(Manganelli & Engle, 2001) classify the Value-at-risk models in three categories:
a) parametric (RiskMetrics, Garch)
b) nonparametric (historical simulation, hybrid model)
c) semiparametric (Extreme Value Theory, CAViaR, quasi-maximum likelihood Garch).

### 2.1. The Parametric models

The RiskMetrics models were the first application of Value-at-risk, developed by J.P. Morgan in 1989, in order to offer a clear position of financial institution regarding their potential losses. Where volatility can be expressed as a function of return and expected value :

$$\sigma_t^2 = (1-\lambda)\sum_{t=1}^{n} \lambda^2 \left(R_t - \mu\right)^2 \quad (2)$$

-where R represent return, $\mu$ is the expected value and $\lambda$ is the exponential factor used in order to quantify the volatility persistent. The $\lambda$ parameter usually has an 0.94 or 0.97 value (Manganelli & Engle, 2001), also the RiskMetrics model assume that the residual are normally distributed.

We will present the parametric models (Garch models) which are the most used because of their capacity to quantify non-linear dependencies. The ARCH models where developed by (Robert F. Engle, 1982) and have the following equations:

$$y_t = B_0 + e_t \quad (3)$$

$$e_t \mid I_t\text{-}1 \sim N(0, ht) \quad (4)$$

$$h_t = ά_0 + ά_1 * e^2_{t-1},\ ά_0 > 0,\quad 0 \le ά1 < 1 \quad (5)$$

The equation (1) expresses the series evolution,where errors are following a normal distribution law (3) of conditional variance define by equations (4). Equations 3 and 4 express the ARCH type models, autoregressive models with different time variance, residuals follow a normal law of 0 mean and ht variance. The value of $ά_0$ and $ά_1$ must be positive, and $ά_1$ has a value between [0,1] in order to avoid an explosive processes, also errors(residuals) follow a normal distribution law.

The GARCH (Generalized autoregressive conditional heteroskedasticity) developed by (Bollerslev, 1986), bring the use of lags as an innovation in equation variance, the equations in the GARCH (1,1) case are:

$$y_t = \beta_0 + e_t \quad (6)$$

$$et \mid It\text{-}1 \sim N(0, ht) \quad (7)$$

$$h_t = ά_0 + ά_1 * e^2_{t-1} + \beta_1 * h_{t-1},\ ά_0 > 0,\quad 0 \le ά_1 < 1 \quad (8)$$

It have been observed that on the financial markets the assets prices are influenced by the news (also called innovation) , so that a bad news generates more volatility than a good news. A GARCH model which treats differently the bad-good news was proposed by (Zakoian, 1994) – Threshold GARCH. It is an asymmetric model in which the conditional volatility is:

$$h_t = ά_0 + ά_1 e^2_{t-1} + \gamma * d_{t-1} * e^2_{t-1} + \beta_1 * h_{t-1} \quad (9)$$

where: $d_t = 1$ if $e_t < 0$ or $d_t = 0$ if $e_t > 0$ .

Also in order to reflect the relation between risk and return another models where proposed in order to incorporate this characteristics (R.F. Engle, Lilien, & Robins, 1987), GARCH in mean model have the following characteristic:

$$y_t = \beta_0 + e_t + \theta * h_t \quad (10)$$

$$et \mid It\text{-}1 \sim N(0, ht) \quad (11)$$

$$h_t = ά_0 + ά_1 * e^2_{t-1} + \beta_1 * h_{t-1},\ ά_0 > 0,\quad 0 \le ά_1 < 1 \quad (12)$$

In this model as the volatility rises the return are rising too, this models are useful in order to capture the risk of the assets.

There is a large variety of GARCH models (Bollerslev, 2008) identifies over 100 types of models, but must of them are variations of some nested models and in the practice the asymmetric , GARCH in mean and the simple GARCH(1,1) models are the most used.

## 4. Conditional Value at Risk (CvaR)

Expected shortfall (ES) or Conditional Value-at-risk (Acerbi & Tasche, 2002) is used as an alternative to Value-at-risk, it can be defined as (Rau-Bredow, 2004): "... the average of all losses which are greater or equal than VaR, the average loss in the worst (1-p)% " and mathematically:

$$ES_\alpha = \frac{1}{\alpha}\int VaR_\gamma(X)d_\gamma \quad (13)$$

where $VaR_\gamma$ is the Value-at-risk and $\alpha$ is the lower quantile.

For example if we invest 1000 in a portfolio and the expected value is presented in column 2 (Portofolio expected value) of Table 1, then the Profit/Loss (column 3) will be the difference between the invested value and the expected value.

***Table 1. Example of expected shortfall calculation***

| Event probability | Portfolio expected value | Profit/loss |
|---|---|---|
| 5% | 100 | -900 |
| 10% | 500 | -500 |
| 20% | 1500 | 500 |
| 50% | 1600 | 600 |
| 15% | 2000 | 1000 |

| Worst case probability $\alpha =$ | Expected shortfall |
|---|---|
| 5% | $ES_{0.05} = -900$ |
| 10% | $ES_{0.10} = \frac{0.05\cdot(-900)+0.05\cdot(-500)}{0.1} = -700$ |
| 20% | $ES_{0.20} = \frac{0.05\cdot(-900)+0.10\cdot(-500)+0.05\cdot(500)}{0.2} = -350$ |
| 50% | $ES_{0.20} = \frac{0.05\cdot(-900)+0.10\cdot(-500)+0.20\cdot(500)+0.15\cdot(600)}{0.5} = 280$ |

ES0.05 represent the worst 5 out of 100 cases, these cases are a subset of the 5% quantile (actually they identify with it) so the expected shortfall is -900, in the case of ES0.20 which represents the worst 20 out of 100 worst cases we calculate the Expected Shortfall as being compose of a 5% quantile with an expected loss of -900 and an 10% quantile with an expected loss of -500 and an 5% quantile with an expected profit of 500.

CvaR or Expected Shortfall are used by (Ferstl & Weissensteiner, 2011) in a multi period stochastic linear programming model with re-allocation, CVaR of shareholder value is minimized in order to , which is the difference between mark to market value of assets and the present value of liabilities.

## 5. Conclusion

In order to apply the management of assets and liabilities in financial institutions it is imperative that the objective function, which usually are the risk level, earnings, liquidity, profit, solvency, the loans and deposits levels, value added

to also take in consideration the worst cases scenario. Value-at-risk and Conditional Value-at-risk can be used as the principal objective function or as a secondary one in ALM. We present the Value-at-risk with the most used models (GARCH) and the Conditional Value-at-risk; the GARCH models can capture the specific of financial series, their asymmetric and leptokurtic characteristics so applying GARCH models in VaR specification will enhance the reliability of forecasts.

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