DECOMPOSING THE GAP IN SCHOOL ACHIEVEMENT BETWEEN FINLAND AND ROMANIA – SOME METHODOLOGICAL ASPECTS

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This paper analyzes the advantages and shortcomings of the Blinder-Oaxaca decomposition. Using PISA data for Finland and Romania, we focus on the drawbacks of the detailed decomposition, when the explanatory variables are categorial. From the best of our knowledge, this kind of analysis is performed for the first time using PISA data.
We show that, using covariates which are categorial, the partial characteristics effects can be different when we use different reference categories of the respective variable. However, the overall characteristics effect of a categorical variable does not depend on the omitted category. The more critical aspect of the interpretation of detailed decomposition, when explanatory variables are categorical, regards the unexplained part of the gap. As we empirically show, the both components of the unexplained part are sensitive to choices of the reference category. These aspects should be taken into account when we perform detailed decompositions with categorial variables.
Keywords: decomposition, categorial variables, test score gap, PISA
JEL Classifications: J24, I21, C14

I. Introduction

In a knowledge based society, skills and knowledge acquired through education play an ever more important role. The economic growth and the development of a country largely depend on the education system. Hence, understanding the differences in school achievement between countries is of huge importance since it allows improving the school systems and, therefore, directly provides relevant information for educational policies.

In this paper we analyze the differences in reading test scores of the students from Finland and Romania. We choose Finland as benchmark for our analysis because it is the best performing country in the PISA study and is considered to have the most effective and equitable school system (Ammermueller 2007). Using PISA (*Programme for International Student Assessment*) data from 2006, we apply a decomposition method widely used in the literature with education data. Based on the example Finland - Romania, our purpose is to focus on the methodological part, showing the advantages and the shortcomings of the method. We refer here to the the *Blinder-Oaxaca decomposition*, which starting with the papers of Blinder (1973) and Oaxaca (1973) has become the most cited decomposition method in applied labor economics. Originally it was designed to decompose and explain wage gaps by sex or race, but in the recent years it was extensively used in decomposing differences in school achievement. A comprehensive overview of the Blinder-Oaxaca decomposition is provided by Fortin, Lemieux, and Firpo (2010).

Krieg and Storer (2006) use the Blinder-Oaxaca method to decompose the differences in test scores between high- and low- performing schools in the state of Washington according to "The No Child Left Behind Act". The motivation of using this method was to determine which factors explain the differences in test scores between schools: the background characteristics of the students or the institutional settings. Ammermüller (2007) uses a slightly different form of the traditional Blinder-Oaxaca decomposition in order to explain the gap in PISA test score between Finland and Germany. The total score gap was decomposed into a characteristics, a return and a characteristics-return effect. The paper reports also these three effects separately for four groups of explanatory variables: student background, resources, institutions and school types. But the results show that, considering only the mean of the distribution, the differences in characteristics could not explain entirely the test scores gap between Finland and Germany. Cho (2007) performs the Blinder-Oaxaca decomposition in order to explain the change in the college enrollments gender gap.

II. Methodology

1. The classical Blinder-Oaxaca Decomposition

The aim of the Blinder-Oaxaca decomposition is to split the differences in mean outcomes between two groups in two components: one, which is attributed to differences in characteristics (endowments) and a component generated by the differences in the regression parameters, given equal endowments. Since we primary focus in this paper on differences in school achievement, the outcomes we refer to are test scores. Formally, the gap in test scores between two groups can be written as:

$$\Delta = E[\mathrm{Y1}|\mathrm{D}\ =\ 1] - \mathrm{E}[\mathrm{Y0}\ |\mathrm{D}\ =\ 0]$$

where D is an indicator of group membership. $D\ =\ 1$ denotes here the group that scores higher, so that the total difference is positive.

The main assumption of the model refers to the structural form of the functions which links the observed and unobserved characteristics in the following way:

$$Y_l\ =\ X\,\beta_l\ +\ \varepsilon_l$$

for $l\ =\ 1, 0$

where $E[\ \varepsilon_l|X\ ]\ =\ 0$.

Using the law of iterated expectations, the gap can be expressed as:

$$\Delta =\ E[X\ |D\ =\ 1]\beta_1\ -\ E[X\ |D\ =\ 0]\beta_0$$

To get the two terms that describe the differences in endowments and the differences in regression coefficients, we must construct a meaningful counterfactual, that excludes general equilibrium effects (Fortin, Lemieux, and Firpo 2010). According to the selected reference group, we add and subtract in the above equation the corresponding counterfactual. In this sense, the assumption about the *nondiscriminatory norm* (Oaxaca 2007), β_0 or β_1, is important, since the decomposition results are not stable to the choice of the reference group *(index number problem*).

In the original specification, Oaxaca (1973) uses as benchmark the more favored group (in terms of nondiscrimination), weighting the differences in average endowments by its coefficient and the coefficient gap by the vector of average characteristics of the more disadvantaged group. In studies analyzing scholar achievement gaps, some authors report the decomposition results from the point of view of the more advantaged group (Krieg and Storer 2006; Duncan and Sandy 2007; Sandy and Duncan 2010; Schneeweis 2010), other studies present the gaps from the point of view of the more disadvantaged group (Cho 2007; Ammermueller 2007; Patacchini and Zenou 2009), while some papers report the results from both specifications (Cook and Evans 2000; McEwan and Marshall 2004).

The gap in test scores in the equation above can be decompose as follows:

$$\begin{aligned}\Delta &=\ E[X\ |D\ =\ 1]\beta_1\ -\ E[X\ |D\ =\ 0]\beta_0\ +\ E[X\ |D\ =\ 1]\beta_0\ -\ E[X\ |D\ =\ 1]\beta_0\\ &=\ E[X\ |D\ =\ 1](\beta_1\ -\ \beta_0) + \ (E[X\ |D\ =\ 1]\ -\ E[X\ |D\ =\ 0])\beta_0\end{aligned}$$

Here we use the counterfactual that describes the average test scores that the individuals from group 1 would get if they would have the returns of group 0. After rearranging the terms, the estimated gap can be decomposed as:

$$\widehat{\Delta}\ = (\bar{X}_1 - \bar{X}_0)\hat{\beta}_0 + \bar{X}_1(\hat{\beta}_1 - \hat{\beta}_0)$$

The right hand side of the equation is due to differences in the average regressors. In economics of education this term is usually called the "explained" or the characteristics effect (Δ_c). It captures the difference of the test scores that would vanish if the individuals from group 0, given their returns to the average characteristics, would have the same characteristics as the individuals from the group 1. In other words this component reflects the gap in test scores that would exist if the students from the both countries were evaluated by the school coun- try from country 0. The second term is due to differences in the coefficients and is named the return effect (Δ_r) or the "unexplained" part of the gap. In studies regarding gaps in outcome between different countries, this term reflects the "country-specific portion" (Zhang and Lee 2011). If it is positive it shows, how much the individuals from the group 0 would on average score better if, given their own characteristics, they would face the returns of group 1.

2. Detailed decomposition

In many studies it is useful to perform a detailed decomposition by looking at the contribution of a certain group of variables or of every single variable to the characteristic and return effect. For example, it would be interesting to see how much of the characteristic effect is due to the differences in the student background or in parents' education alone. Performing a variable-by-variable decomposition, the effects of the total gap can be written as the sum over the covariates:

$$\hat{\Delta}_c = \sum_{k=1} (\bar{X}_{1k} - \bar{X}_{0k})\hat{\beta}_{0k}$$

$$\hat{\Delta}_r = (\hat{\beta}_1 - \hat{\beta}_0) + \sum_{k=1} \bar{X}_{0k}(\hat{\beta}_{1k} - \hat{\beta}_{0k})$$

$(\bar{X}_{1k} - \bar{X}_{0k})\hat{\beta}_{0k}$ represents the component of the characteristics effect that is described by the differences in the average characteristics of the k^{th} covariate, multiplied by the respective $\hat{\beta}$. $\hat{\beta}_1 - \hat{\beta}_0$ is the portion of the unexplained gap due to group membership (Jones and Kelley 1984). The differences in the returns to the k^{th} covariate, evaluated at the average value of the characteristics X_k $[\bar{X}_{0k}(\hat{\beta}_{1k} - \hat{\beta}_{0k})]$ represents the k^{th} component of the return effect.

In the field of education it could be informative to separate the contribution of variables that describe the student background, for example, to the characteristic effect from the contribution of other variables related to the resources, institutional aspects etc. Furthermore, the contribution of individual variables to the total gap can be detected performing a more detailed decomposition.

Zhang and Lee (2011), for example, decompose the PISA 2006 test gap in math scores for a group of OECD countries. Calculating the gap in test scores as the difference between each individual country and the OECD average, they apply a *Blinder-Oaxaca detailed decomposition* using different blocks of variables, namely demographic characteristics,study time and activities, family background, and school characteristics.

Ammermueller (2007) performs a similar detailed decomposition for differences in reading test scores between Finland and Germany, dividing the gap into three components: a characteristic (endowment) effect, return effect and a component reflecting the interaction between the first two effects. The explanatory variables are grouped in four categories: student background, resources, institutions and school types.

Duncan and Sandy (2007) analyze the achievement test score gap between public and private schools. Based on the Blinder-Oaxaca technique, they perform a detailed decomposition, calculating the explained and unexplained effects both at the level of each explanatory variable, as well as for the groups of some variables, that describe the family background and school quality. In a paper from 2010 (Sandy and Duncan 2010), they apply also a variable-by-variable decomposition in order to identify the sources of the test score gap between urban and suburban schools.

III. Critical aspects regarding the detailed Blinder-Oaxaca decomposition

Beyond the relevant information that a detailed decomposition can offer, the interpretation of the estimated coefficients can be misleading, especially when the covariates are categorical, with more than two categories (Fortin, Lemieux, and Firpo 2010). In order to illustrate this aspect, we use the PISA test score gap in reading between Finland and Romania. The data contain many relevant variables that are categorical: parents' education (no secondary education, lower and upper secondary education, tertiary education), the number of books at home (less than 10 books, between 11-50, 51-100, 101-250, 251-500 and more than 500 books) etc.

Table 1 presents the estimated coefficients of the regression of the reading score on the individual (gender, age (in months), grade) and student background variables for the Romanian students (the reference group in the decomposition). We look at the results of mother's education and of number of books. In column 2, mother's tertiary education and *book6* are the omitted categories, while in column 4 we use the level of no secondary education and book1 as the reference categories, so that for the categorical variables the estimated coefficients in the first regression are equal to the corresponding coefficients in the second regression minus the value of the coefficient of the variable, which in the first regression is the omitted one (see note 1).

Table 1: *OLS Regression Coefficients for Romanian Students*

Variable	First regression		Second regression	
	Coefficient	(Std. Error)	**Coefficient**	(Std. Error)
Dmale	-42.01	(2.14)	-42.01	(2.14)
age m	-1.05	(0.33)	-1.05	(0.33)
grade8	108.53	(13.38)	108.53	(13.38)
grade9	121.33	(13.15)	121.33	(13.15)
grade1011	156.71	(14.52)	156.71	(14.52)
book1	-88.02	(5.39)	-	-
book2	-59.37	(5.15)	28.65	(3.60)
book3	-33.25	(4.91)	54.77	(3.38)
book4	-15.75	(5.22)	72.27	(3.96)
book5	6.41	(5.47)	94.43	(4.55)
book6	-	-	88.02	(5.39)
m nosec	-14.91	(7.60)	-	-
m lower sec	-16.12	(4.63)	-1.21	(7.92)
m upper sec	7.99	(2.82)	22.90	(7.28)
m ter	-	-	14.91	(7.60)
f nosec	-0.22	(7.62)	-0.22	(7.62)
f lower sec	-23.01	(5.25)	-23.01	(5.25)
f upper sec	-10.76	(2.93)	-10.76	(2.93)
Intercept	539.77	(64.12)	436.62	(64.31)

Source: PISA 2006 data, own calculations. Standard errors are in brackets.

Based on the results from both regressions in Table 1, we can compare for each category, which are not omitted in any of the both regressions, the magnitudes of the characteristics effects. Comparing the results from column 2 and 4, we can argue that having in Romania more students whose mothers have an upper secondary education level than in Finland, the characteristics effect of this variable is almost three times as large (22.90) in one case (column 4) as in the other case (7.99) (column 2). Using the same pattern to interpret the estimates regarding the variable *book5*, we can argue that the corresponding partial characteristics effect is fourteen times larger in the second case (column 4) as in the first case (column 2). But this interpretation should be viewed as caution, since in the two cases the returns are obtained relative to different reference categories of the respective variable. This issue is especially problematic when the categorical variables do not have an absolute interpretation (Fortin, Lemieux, and Firpo 2010).

Even if the differences in the partial effects between columns (2) and (4) are large, the overall contribution of mother's education and of number of books to the characteristic effect is the same in both cases, as can be seen in the Table 2. Thus, the overall characteristics effect of a categorical variable does not depend on the omitted category. We also show this formally.

Suppose that we have in our regressions an explanatory variable with four categories ($k = a, b, d, c$), from which one of them is the reference category and this variable is the only covariate in the regression. We perform two regressions, one in which the first category is the omitted one and then, in the second regression, we omit the 4^{th} category. We denote by $\gamma_{l,k}$ the coefficients of the first regression ($\gamma_{l,a} = 0$) and by $\beta_{l,k}$ the coefficients of the second regression, where $\beta_{l,d} = 0$. The overall characteristics effect corresponding to the first regression can be written as follows:

$$\sum_{k \neq a} (\bar{X}_{1,k} - \bar{X}_{0,k}) \hat{\gamma}_{0,k}$$

Using the fact that $\hat{\gamma}_{0,k} = \hat{\beta}_{0,k} - \hat{\beta}_{0,a}$ (see note 1) and $\sum_{k \neq a} \bar{X}_{l,k} = \sum_k \bar{X}_{l,k} - \bar{X}_{l,a}$, we get:

$$\sum_{k \neq a} (\bar{X}_{1,k} - \bar{X}_{0,k}) \hat{\gamma}_{0,k}$$

$$= \sum_{k \neq a} (\bar{X}_{1,k} - \bar{X}_{0,k}) (\hat{\beta}_{0,k} - \hat{\beta}_{0,a})$$

$$= \textstyle\sum_{k \neq a} (\bar{X}_{1,k} - \bar{X}_{0,k})\ \hat{\beta}_{0,k} - \hat{\beta}_{0,a} (1 - \bar{X}_{1,a}) + \hat{\beta}_{0,a} (1 - \bar{X}_{0,a})$$

$$= \textstyle\sum_k \bar{X}_{1,k} - \bar{X}_{0,k} \hat{\beta}_{0,k}$$

The more critical aspect of the interpretation of detailed decomposition, when explanatory variables are categorical, regards the unexplained part of the gap. As showed above, the unexplained component is the sum between two differences: one between the intercepts and the other between the coefficients. Performing a detailed decomposition when different categories of a variable represent the reference categories, the values of the both components of the unexplained part are not the same in each case. This fact is illustrated in Table 2.

When the omitted categories are *book6* and *mother's tertiary education*, the effects which incorporate the difference in the coefficients are −13.36 for the *number of books*, and −6.41 for *mother's education* compared to −24.33 and −13.18 in the second case (*book1* and *mother's no secondary education* are omitted). The rest of the unexplained gap of these two variables is captured by the intercepts, so that the total unexplained part of the gap is the same in both cases (137.31).

Due to the fact that the both components of the unexplained part are sensitive to choices of the reference category, some studies about school achievement gap treats the unexplained part as a residual and don't interpret the results of the detailed decomposition for this component (McEwan and Marshall 2004; Sandy and Duncan 2010).

Table 2: *Oaxaca-Blinder Detailed Decomposition Results*

Variable	First case		Second case	
	Coefficient	(Std. Error)	**Coefficient**	(Std. Error)
Equation 1: Differential				
Prediction_1	548.50	(1.13)	548.50	(1.13)
Prediction_2	396.01	(1.66)	396.01	(1.66)
Difference	152.49	(2.01)	152.49	(2.01)
Equation 2: Explained				
Dmale	0.15	(0.53)	0.15	(0.53)
age m	1.20	(0.51)	1.20	(0.51)

grade8	-2.59	(1.17)	-2.59	(1.17)
grade9	7.10	(1.66)	7.10	(1.66)
grade1011	-4.53	(0.83)	-4.53	(0.83)
books	13.69	(1.00)	13.69	(1.00)
Motherduc	-1.35	(0.96)	-1.35	(0.96)
f nosec	-0.01	(0.40)	-0.01	(0.40)
f lower sec	-0.45	(0.23)	-0.45	(0.23)
f upper sec	1.97	(0.78)	1.97	(0.78)
Total	15.18	(1.49)	15.18	(1.49)
Equation 3 : Unexplained				
Dmale	-0.07	(1.71)	-0.07	(1.71)
age m	108.38	(102.63)	108.38	(102.63)
grade8	-5.79	(3.21)	-5.79	(3.21)
grade9	-27.69	(24.93)	-27.69	(24.93)
grade1011	-0.01	(0.01)	-0.01	(0.01)
books	-13.36	(7.01)	-24.33	(6.06)
motherseduc	-6.41	(2.02)	-13.18	(11.94)
f nosec	-0.54	(0.76)	-0.54	(0.76)
f lower sec	1.16	(0.78)	1.16	(0.78)
f upper sec	0.28	(2.04)	0.28	(2.04)
Intercept	81.36	(106.36)	99.09	(105.68)
Total	137.31	(2.01)	137.31	(2.01)

Source: PISA 2006 data, own calculations. Standard errors are in brackets.

IV. Conclusions

This paper analyzes the advantages and shortcomings of the Blinder-Oaxaca decomposition. Using PISA data for Finland and Romania, we show especially the drawbacks of the detailed decomposition, when the explanatory variables are categorial. From the best of our knowledge, this kind of analysis is performed for the first time using PISA data.

We showed that, using covariates which are categorial, the partial characteristics effects can be different when we use different reference categories of the respective variable. However, the overall characteristics effect of a categorical variable does not depend on the omitted category.

The more critical aspect of the interpretation of detailed decomposition, when explanatory variables are categorical, regards the unexplained part of the gap. As we empirically showed, the both components of the unexplained part are sensitive to choices of the reference category.

These aspects should be taken into account when we perform detailed decompositions with categorial variables.

Notes

1. Suppose we have the following model, corresponding to group 0: $Y_i = \gamma_{0,a} + \gamma_{0,b} D_{bi} + \gamma_{0,c} D_{ci} + \gamma_{0,d} D_{di} + u_\gamma$, where D_{ki} represent the dummy variables for the categories k, the category a being the reference one. When d is the base category, the model is the following:

$Y_i =$
$\beta_{0,d} + \beta_{0,a} D_{ai} + \beta_{0,b} D_{bi} + \beta_{0,c} D_{ci} +$
$+ u_\beta =$

$kicategory\ s\ in\ Table\ (3.1)es, test\ score\ gap, 170170170170170170170170170$ $+$
$170170170170170170170170170170170170170170170170170170170170170170170\beta_{0,d}$

$\beta_{0,a}\,(1 - D_{bi} - D_{ci} - D_{di}) + \beta_{0,b}\, D_{bi} + \beta_{0,c}\, D_{ci} + u_\beta = \beta_{0,d} + \beta_{0,a} + D_{bi}\,(\beta_{0,b} - \beta_{0,a}) + D_{ci}(\beta_{0,c} - \beta_{0,a}) + u_\beta$. $E(Y_i|D_{ki} = 1) - E(Y_i|D_{ki} = 0) = \beta_{0,k} - \beta_{0,a} = \gamma_{0,k}$.

2. ACKNOWLEDGEMENT: This paper was made within The Knowledge Based Society Project supported by the Sectoral Operational Programme Human Resources Development (SOP HRD), financed from the European Social Fund and by the Romanian Government under the contract number POSDRU ID 56815.

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