MULTIFRACTAL STRUCTURE OF CENTRAL AND EASTERN EUROPEAN FOREIGN EXCHANGE MARKETS

Trenca Ioan
Babeş-Bolyai University, Faculty of Economics and Business Administration

Pleşoianu Anita
Babeş-Bolyai University, Faculty of Economics and Business Administration

Căpuşan Răzvan
Academy of Economic Studies, Faculty of Finance, Insurance, Banking and Stock Exchange

It is well known that empirical data coming from financial markets, like stock market indices, commodities, interest rates, traded volumes and foreign exchange rates have a multifractal structure. Multifractals were introduced in the field of economics to surpass the shortcomings of classical models like the fractional Brownian motion or GARCH processes.

In this paper we investigate the multifractal behavior of Central and Eastern European foreign exchange rates, namely the Czech koruna, Croatian kuna, Hungarian forint, Polish zlot, Romanian leu and Russian rouble with respect to euro from January 13, 2000 to February 29, 2012. The dynamics of exchange rates is of interest for investors and traders, monetary and fiscal authorities, economic agents or policy makers. The exchange rate movements affect the international balance of payments, trade flows, and allocation of the resources in national and international economy. The empirical results from the multifractal detrending fluctuation analysis algorithm show that the six exchange rate series analysed display significant multifractality. Moreover, generating shuffled and surrogate time series, we analyze the sources of multifractality, long-range correlations and heavy-tailed distributions, and we find that this multifractal behavior can be mainly attributed to the latter. Finally, we propose a foreign exchange market inefficiency ranking by considering the multifractality degree as a measure of inefficiency. The regulators, through policy instruments, aim to improve the informational inefficiency of the markets, to reduce the associated risks and to ensure economic stabilization. Evaluation of the degree of information efficiency of foreign exchange markets, for Central and Eastern Europe countries, is important to assess to what extent *these countries are prepared for the transition towards fully monetary integration. The weak form efficiency implies that the past exchange rates cannot help to improve forecasts about future spot exchange rates, therefore there are no opportunities* for profit *based upon past data. Our results show that the Russian foreign exchange market has the highest degree of efficiency while the Hungarian foreign exchange market is at the opposite side.*

Keywords: multifractality, MFDFA, long-range correlations, heavy-tailed distributions, foreign exchange markets.

JEL Codes: C10, F31, G15.

I. Introduction

The financial markets are open and complex dynamical systems with a large number of interacting elements. For institutional investors, a correct assessment of these systems is very important to determine the optimal investment strategy. Recently, the analysis of financial markets has been found to exhibit some universal characteristics similar to those observed in physical systems with a large number of interacting units. Since the classical financial theory cannot explain effectively this characteristics, (multi)fractal theoretical models were introduced in the field of economics by Mandelbrot (1997) to study the economical and financial data from a new perspective.

A fractal is an object that displays self-similarity on all scales. Self-similarity means that each piece can be considered a reduced-scale image of the whole. Mandelbrot (1977) uses the characteristic of self-similarity to develop the concept of fractional dimension, not integer. "When the dimension of a time series is non-integer, this is associated with two specific features: inhomogeneity - extreme fluctuations at irregular intervals, and scaling symmetries - definite

relationships between fluctuations over different separation distances" (Scarlat, Stan and Cristescu 2007:188-189).
In existing literatures, the presence of fractal characteristics in financial markets implied the market complexity (Jiang and Zhou 2009), crash predictions (Grech and Pamula 2008), volatility predictability (Wei and Wang 2008) and inefficiency (Cajueiro and Tabak 2004, 2007). Among the techniques of fractal analysis on financial time series are those based on Hurst exponent (H). In the case of unifractal processes, the scaling behavior is determined from a unique constant H, while in the case of multifractal processes a continuous spectrum of exponents is needed.
The originality of this research is given by several aspects. First, is the first study that compares the multifractal behavior of six Central and Eastern European foreign exchange rates, namely the Czech koruna (CZH), Croatian kuna (HRK), Hungarian forint (HUF), Polish zlot (PLN), Romanian leu (RON) and Russian rouble (RUB) with respect to euro (EUR) from January 13, 2000 to February 29, 2012 via a robust and powerful technique – the Multifractal Detrended Fluctuation Analysis (MFDFA). The exchange rates between currencies are a particularly interesting category of economic data to study as they dictate the economy of most countries. Second, we quantify the contributions of two different factors that are thought to be the causes of multifractality – long-range correlations of the time series and heavy-tailed distributions – by generating shuffled and surrogated time series form the original ones. Third, we follow Zunino et al. (2009) and we build an inefficiency ranking by considering the multifractality degrees as a measure of inefficiency.

II. Literature review

Multifractal analysis which was initially employed to investigate the intermittent nature of turbulence has been widely applied to financial time series including stock prices, interest rates, commodity prices and exchange rates. For example, Ausloos (2000), using the DFA technique, empirically shows that several foreign exchange rate series display significantly long-range persistently or anti-persistently autocorrelated behavior. Using R/S analysis, Kim and Yoon (2004) find the multifractal behaviors in the Korea Won–USD and Japan Yen–USD exchange rates. Norouzzadeh and Rahmani (2006) study the dynamics of Iranian Rial–US dollar exchange rates and find the evidence of multifractality based on MFDFA. Moreover, they show that the main source of multifractality are long-range correlations of small and large fluctuations. The same source of multifractality is found by Wang et al. (2011) in case of twelve exchange rates series. For USD/AUD, USD/EUR and CNY/USD, both fat-tail distribution and long-range correlations have important contributions to the multifractality. In addition, the authors find that extreme events play an important role in the contributions to multifractality of the USD/EUR exchange rate series. Tabak and Cajueiro (2006) estimate Hurst exponents using the local Whittle estimator and find that the Euro bilateral exchange rates of the US, Canadian and Singapore Dollar are amongst the most efficient currencies, while Japanese Yen and Swedish Krona are amongst the most inefficient. Jiang, Ma and Cai (2007) show that distribution of relative return of 74 global currencies is a power-law and the distribution of correlation coefficients in terms of relative return in currency price changes is time dependent, based on the method of scaled factorial moment. The correlation of foreign exchange rates and their degree of asynchrony is investigated by Liu, Qian and Heng (2010) for a period from 1995 to 2002. The results show that the cross-sample entropy of every two exchange rates returns of DKK/USD, NOK/USD, CAD/USD, JPY/USD, KRW/USD, SGD/USD, THB/USD and TWD/USD become higher after the Asian currency crisis, indicating a higher asynchrony between the exchange rates. Wang, Yu and Suo (2012) provide evidence that the change in the yuan exchange rate regime in July 2008 caused different multifractal properties of CIB/CNY Composite Index in 2008-2010 compared to 2005-2008. The Romanian currency market drew the attention of Scarlat, Stan and Cristescu (2007). The authors observed that due to the "in-phase" evolution of the economic agents, the statistical self-similarity of the time series of the daily exchange rate RON/USD resembles a theoretical self-similarity. The self-similar cells of dimensions obeying a definite power law scaling rule are related to five categories of economic agents detected via a crossing-type analysis based on the Hurst exponent and the frequency spectrum and are employed in the study of the fragmentation-defragmentation process.

III. Methodology

The MFDFA is a generalization of the detrended fluctuation analysis (DFA) and it was developed by Kantelhardt et al. (2002) for non-stationary multifractal data. The procedure can be described as follow:

- determine the "profile" $Y_j = \sum_{i=1}^{j}\left(x_i - \langle x\rangle\right)$, where x_i is a series of lenght N and $\langle x\rangle$ denotes the averaging over the whole time series, and divide it into $N_s = \mathrm{int}(N/s)$ nonoverlapping segments of size s;
- calculate the local trend for each segment ν by the least square fit of the series and then determine the variance $F_\nu^2(s) = 1/s\sum_{j=1}^{s}\{Y[(\nu-1)s+j]-y_\nu(j)\}^2$ between the local trend and the profile in each segment ν. Here, $y_\nu(j)$ is the fitting polynomial in the $\nu-\mathrm{th}$ segment;
- average over all segments to obtain the q-th order fluctuation function, defined as $F_q(s) = \left\{1/2N_s\sum_{\nu=1}^{2N_s}\left[F_\nu^2(s)\right]^{q/2}\right\}^{1/q}$. In general, $F_q(s)$ scales with s as $F_q(s) \approx s^{H(q)}$ with the generalized Hurst exponent *H(q)*. For a monofractal time series, *H(q)* is independent of q and the scaling behavior of $F_\nu^2(s)$ is identical for all values of q. For multifractal data, *H(q)* depends on the chosen moment q since the small and large fluctuations scale in a different way. When q has a large positive value, *H(q)* implies large scale fluctuations. By contrast, if q is negative or has a very small positive value, *H(q)* describes small scale fluctuations. Therefore, richer multifractality corresponds to higher variability of *H(q)*. We follow Zunino et al. (2009) and we quantified the multifractality degree and hence we define our measure of inefficiency by

$$Inefficien\,cy = \Delta H = H(q_{\min}) - H(q_{\max}) \tag{1}$$

In order to determine the contributions of long-range temporal correlations and heavy-tailed probability distributions of variations in the multifractality we follow the procedures introduced by Norouzzadeh and Rahmani (2006): shuffling (1) and Fourier phase randomization (2). First, we destroyed any temporal correlations by randomly shuffling the return time series and we quantified the influence of correlation by

$$H_{cor}(q) = H(q) - H_{shuf}(q) \tag{2}$$

Second, since the shuffling procedure doesn't affect the probability density function (PDF), we create phase-randomized surrogate time series to study the contribution of the heavy-tailed distribution. Thus, the contribution of the non-Gaussian PDF will be

$$H_{PDF}(q) = H(q) - H_{sur}(q) \tag{3}$$

IV. Empirical results

Fig. 1 illustrates the generalized Hurst exponents *H(q)* via the MFDFA procedure for the original, shuffled and surrogate daily exchange rate returns of the six markets investigated. In our study we follow Zunino et al. (2009) and use a polynomial fit of order 3 (hence, the algorithm is denoted as MFDFA-3) and window lengths (s) between 20 and N/4 with a step of 4 according to the suggestions of Kantelhardt et al. (2002). The variation of q is from [-10, 10] with a step of 1.
Overall, it can be observed, especially in case of original returns, that the change of *H(q)* depends on the values of q indicating that the EUR exchange rate series are multifractal. The

highest variability of *H(q)* is displayed by HUF/EUR exchange rate, followed by RON/EUR and HRK/EUR exchange rates. It should be mention that for most series, *H(q)* monotonically decreases with the value of *q* increases, except the surrogate return series.

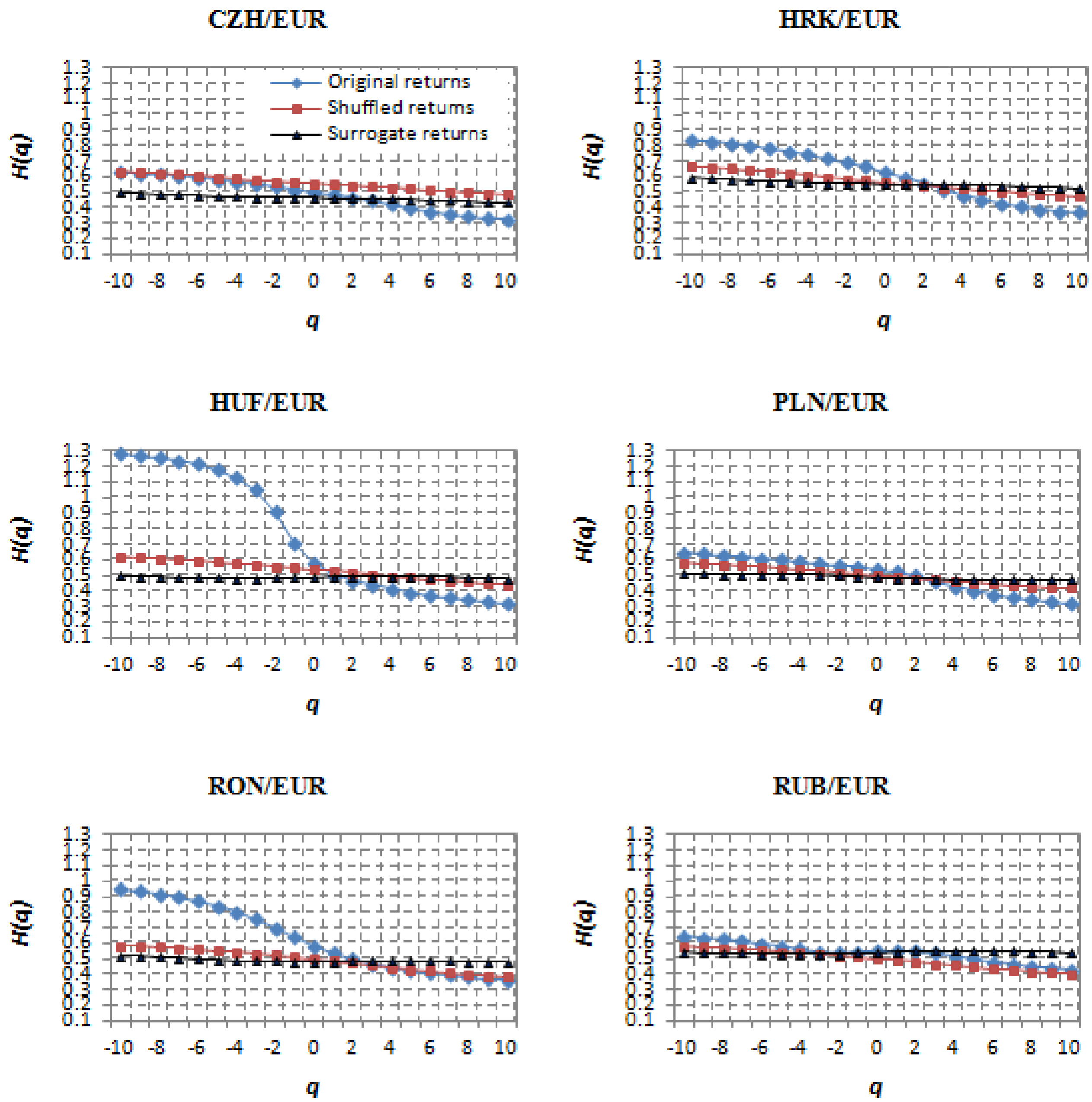

Fig. 1. *Generalized Hurst exponent, H(q), as a function of q for the original, shuffled and surrogate daily exchange rate returns*

Source: authors' calculations.

Multifractality degrees of the original, shuffled and surrogate return series are reported in Table 1. As expected, the associated shuffled and surrogate time series have a lower degree of multifractality than the original time series ($\Delta H_{shuf} < \Delta H$, $\Delta H_{sur} < \Delta H$) indicating that both long memory and heavy-tailedness have major contributions to the multifractality. However, in all cases, the multifractality degrees of surrogate series weakened more remarkably that those of the shuffled series ($\Delta H_{sur} < \Delta H_{shuf}$) pointing out that heavy-tailed distributions play a more significant role in the sources of the observed multifractality ($\Delta H_{PDF} > \Delta H_{cor}$).

Table 1. *Degrees of multifractality for the daily exchange rate returns*

Exchange rate	ΔH	ΔH_{shuf}	ΔH_{sur}	ΔH_{cor}	ΔH_{PDF}
CZH/EUR	0.305	0.146	0.065	0.159	0.240
HRK/EUR	0.466	0.191	0.069	0.274	0.396
HUF/EUR	0.963	0.179	0.015	0.784	0.948
PLN/EUR	0.323	0.162	0.039	0.161	0.284
RON/EUR	0.585	0.204	0.042	0.381	0.542
RUB/EUR	0.217	0.181	0.005	0.036	0.212

Source: authors' calculations.

In Table 2 we rank the exchange rate returns by considering their multifractality degree as a measure of inefficiency. Ranking market inefficiency is important information for investors and for regulatory authorities and has implications for financial theories and investment strategies. The multifractality degree is estimated as in Eq. 1 and also by the mean absolute deviation of the generalized Hurst exponents. The second measure is chosen because it is independent of the Gaussian assumption like the standard deviation. As one can observe comparing the first and the third column of Table 2, the ranking order of the foreign exchange markets is unchanged. In the first position in the ranking is the RUB/EUR exchange rate, while the last position is occupied by the HUF/EUR exchange rate. Thus, the Russian foreign exchange market has the highest degree of efficiency, the returns of the RUB/EUR exchange rate being less predictable than those of HUF/EUR.

Table 2. *Inefficiency ranking of daily exchange rate returns*

Measure: multifractality degree [Eq (1)]		**Measure: mean absolute deviation of the *H(q)***	
RUB/EUR	0.217	RUB/EUR	0.051
CZH/EUR	0.305	CZH/EUR	0.093
PLN/EUR	0.323	PLN/EUR	0.101
HRK/EUR	0.465	HRK/EUR	0.153
RON/EUR	0.585	RON/EUR	0.201
HUF/EUR	0.963	HUF/EUR	0.371

Source: authors' calculations.

V. Conclusions

In this paper, we firstly studied the multifractal properties of six Central and Eastern European foreign exchange markets through the MFDFA technique. The main advantage of this technique is that it can be used for nonstationary multifractal data. Basic notion is the examination of deviations from polynomial fit of different moments q. We show that the time series for exchange rate variations exhibit the characteristics that can be interpreted in terms of multifractality. The highest variability of *H(q)* is displayed by HUF/EUR exchange rate, followed by RON/EUR and HRK/EUR exchange rates. Second, we employ the shuffling and Fourier phase randomization procedures to obtain the origins of the multifractality. It has been shown that there are two main factors leading to multifractal behavior of financial time series, nonlinear time correlations between present and past events and the heavy-tailed probability distributions of function. Our results provide the evidence that the multifractality can be mainly attributed to the fat-tail distributions and secondarily to the long-range correlations, since the degree of multifractality of the surrogate return series became significantly weaker than that of the shuffled series. Moreover, we follow Zunino et al. (2009) and we introduce the multifractality degree as a measure of inefficiency and rank the six foreign exchange rates. Two different estimators for the multifractality degree were introduced: the range and the mean absolute deviation of the generalized Hurst exponents and the same results were obtained in both approaches: the RUB/EUR exchange rate is the less predictable, while the HUF/EUR exchange rate is the most predictable.

VI. Notes

(1). Shuffling procedure preserves the distribution of the variations but destroys any temporal correlations. What then remains are data with exactly the same fluctuation distributions but without memory. For more details see Norouzzadeh and Jafari (2005).

(2). Phase randomization preserves the amplitudes of the Fourier transform but randomizing the Fourier phases. This procedure eliminates nonlinearities, preserving only the linear properties of the original time series. For more details see Norouzzadeh and Jafari (2005).

VII. References

Ausloos, Marcel. “Statistical physics in foreign exchange currency and stock markets”. *Physica A* 285 (2000): 48-65.

Cajueiro, Daniel and Tabak, Benjamin. “The Hurst exponent over time: testing the assertion that emerging markets are becoming more efficient”. *Physica A* 336 (2004): 521-537.

Grech, Dariusz and Pamula, Grzegorz. “The local Hurst exponent of the financial time series in the vicinity of crashes on the Polish stock exchange market”. *Physica A* 387 (2008): 4299-4308.

Jiang, J., Ma, K. and Cai, X. “Scaling and correlations in foreign exchange market”. *Physica A* 375 (2007): 274-280.

Jiang, Zhi-Qiang and Zhou, Wei-Xing. “Detrended fluctuation analysis of intertrade durations”. *Physica A* 388 (2009): 433-440.

Kantelhardt, Jan, Zschiegner, Stephan, Koscielny-Bunde, Eva, Havlin, Shlomo, Bunde, Armin and Stanley, Eugene. “Multifractal detrended fluctuation analysis of nonstationary time series”. *Physica A* 316 (2002): 87-114.

Kim, Kyungsik and Yoon, Seong-Min. “Multifractal features of financial markets”. *Physica A* 344 (2004): 272-278

Liu, Li-Zhi, Qian, Xi-Yuan and Heng-Yao, Lu. “Cross-sample entropy of foreign exchange time series”. *Physica A* 389 (2010): 4785-4792.

Mandelbrot, Benoît. *Fractals and Scaling in Finance*, New York: Springer, 1997.

Mandelbrot, Benoît. Fractals: *Form, Chance and Dimension*, San Francisco: Freeman, 1977

Norouzzadeh, P. and Rahmani, B. “A multifractal detrended fluctuation description of Iranian rial-US dollar exchange rate”. *Physica A* 367 (2006) 328-336.

Scarlat, Eugen, Stan, Cristina and Cristescu, Constantin. “Self-similar characteristics of the currency exchange rate in an economy in transition”. *Physica A* 370 (2007): 188-198.

Tabak, Benjamin and Cajueiro, Daniel. “Are the crude oil markets becoming weakly efficient over time? A test for time-varying long-range dependence in prices and volatility”. *Energy Economics* 29 (2007): 28-36.

Tabak, Benjamin and Cajueiro, Daniel. “Assessing inefficiency in euro bilateral exchange rates”. *Physica A* 367 (2006): 319-327.

Wang, Dong-Hua, Yu, Xiao-Wen and Suo, Yuan-Yuan. “Statistical properties of the yuan exchange rate index”. *Physica A* 391 (2012): 3503-3512.

Wang, Yudong, Wu, Chongfeng and Pan, Zhiyuan. “Multifractal detrending moving average analysis on the US Dollar exchange rates”. *Physica A* 390 (2011): 3512-3523.

Wei, Yu and Wang, Peng. “Forecasting volatility of SSEC in Chinese stock market using multifractal analysis”. Physica A 387 (2008) 1585-1592.

Zunino, Luciano, Figliola, Alejandra, Tabak, Benjamin, Perez, Dario, Garavaglia, Mario and Rossi, Osvaldo. “Multifractal structure in Latin-American market indices”. Chaos, Solitons and Fractals 41 (2009): 2331-2340.