SUBSTANTIATION OF THE PUBLIC DEBT SUSTAINABILITY USING KALMAN FILTER

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Global economic conditions have pushed many countries into the delicate situation of contracting foreign loans, leading overnight at alarming volumes of public debt. The need for control and relevant analysis for the sustainability of a country's public debt has led us to use the Kalman filter in predicting future values of the key indicators of public debt. The development of a mathematical model of analysis for public services and the budget deficit was necessary to objectively assess the level of the public debt sustainability.Knowing future values of the public debt or the future evolutions of the revenues for the operational budget, offers the posibility of a better handling of the operational expenditures and finally a better balance for the public budget deficit.Using the mathematical mechanism of Kalman filters implemented in Matlab programming language, we generated the estimated future values of the proposed model proposed and key indicators, the results confirming the fears of a low public debt sustainability for Romania.We predicted the future values for the debt service, the public external debt and the operational public revenues,expenditures and deficit, and compared them, to obtain an image of the future evolution and position of the sustainability of the public debt. The work in this paper is an innovative one for the public science sector, and the results obtained are promising for future researches. The values estimated by the Kalman filter are an orientation for the future public policies, and indicate a rather stable but negative evolution for the public debt service. The sustainability of the public debt depends on the decisions taken for the correction of the estimated values, in changing the negative evolution of the budgetary indicators into a positive one.Taking all this into consideration we will conclude that the mathematical mecanism of the Kalman filters offers valuable informations for Government and future research should be oriented to develop it's returned results.

Keywords: Kalman filter, debt, sustainability, deficit, prediction.

JEL Codification: H63, H68, C02, C53, C88

I. Introduction

The financial crisis, transformed almost overnight into the global economic crisis, gave serious headaches to the central governments concerned with the reduction of the public budget deficits that the economic crisis has left behind. Although some economists (Nouriel Roubini(1)) have provided since before a housing price drop, a brutal shock of oil prices, a dramatic drop in the consumer's confidence, and inevitably a crisis, they were not taken into account. What was almost incomprehensible about the financial crisis proved to be a shocking reality shortly. A tough warning from the economists has become reality before long.

Many of those who saved, once the crisis began saw their dreams to gain access to education or to benefit from pension insurance shattered. With the financial crisis the global economy fell,

financial economics and central governments were faced with huge budget deficits that were threatening the national security of states.
The consequence of global economic crisis on the public budget was immediate. Business activities have suffered contractions and with this the budget revenues began to decrease dramatically. Consumers have abandoned durable goods, and reduced their consumption of utilities and remained at emergency levels of consumption goods for food and clothing. The times were almost apocalyptic for national economies, especially since governments have had to take a series of measures to delay public spending.
What did the central government to these financial hurricanes that have had to face?
Firstly., consider the case of Romania. Central Government faced a slump in income from the budget sought to adopt a strategy of rationalization of public expenditure in two ways. The first way was to reduce by 25 percent the salaries in the public sector. With all the strength that unions have expressed the measure was implemented and the effects are felt today in the public sector. Only 15% from the reduction of public sector wages that has been applied with the start of the crisis, were recovered. A second way, similar to the first, was also in rationalizing expenditure, so that the central government tried to aprove the reduction with 15 percent of the pensions. But Romania's Constitutional Court rejected the government's decision and considered it as unconstitutional. So this measure has been replaced with a last resort one, the increase of VAT with 5 percent. It remains to be seen whether this measure has proved to be a rational from the perspective of increasing public government revenues or the measure will impact on the long term, the individual consumption of the population. Whatever the hedging strategies for covering the huge budget deficits that were triggered by the global economic crisis, government should identify the measures necessary to prevent insolvency for its own employees or even worse, to its financial creditors.
Moreover, as the mentioned measures were not sufficienct, the government decided to contract a loan from the International Monetary Fund worth 20 billion euros, known at that time as Romania’s belt. The declared purpose of the loan was to cover the budget deficit and to strengthen Romania's foreign reserve.
The reality is that the national economy was in need of strengthening it’s foreign exchange reserves to mitigate pressures on the exchange rate. So to avoid another financial risk that could embarrass macroeconomic conditions triggered by global economic crisis, namely the risk of uncontrolled growth of foreign exchange. It would inevitably affect the consumers whom could not exert influence on the growth of consumer’s goods prices.
Threats of the huge budget deficits, the rising of the public debt (almost exponentially) and the uncontrolled increase in the exchange rate are just some of the unwanted consequences of the global economic crisis.
How have other states responded to these threats? Overall the recipe was pretty much the same. Streamlining personnel expenditures (reduction with 2-3 percent for Germany, Italy and France) followed by a sustaining of the capital expenditures are the steps that states followed not to be affected from the huge budget deficits. Moreover, the ultimate saving came from the International Monetary Fund, which conditioned the public loans to cover budget deficits, with the the implementation of fiscal policy measures that have often proved unpopular. It is also interesting the case of Hungary, which identified the public service monopolies and financial institutions (banks) as a source of additional revenue to the public budget. In this way, additional profits were taxed from the banks and from the state companies operating in monopoly markets, particularly the gas market and electricity market.
It is obvious that all central governments saw public borrowing as the easiest path to follow for avoiding the risk of default, or to avoid cash shortages which threatened more than ever the public institutions.

The explanation is simple: You can not stimulate economic growth in order to collect additional revenue to the public budget by supporting short-term measures. This requires a long-term fiscal and budgetary strategy, including support for public investment, leading ultimately to economic growth necessary to supplement revenue to the public budget.
But public debt and debt service raises serious problems resulting from its sustainability in the budget. A simple rule shows that debt sustainability should be supported from the outturn of the operational budget. This means that the current revenues of the budget, after covering the operational expedintures are to cover the public debt service. But the current public budget revenues depend on two variables namely the impact of fiscal policies that support the central government in general and economic growth recorded by the national economies.
Sound fiscal policies of automatic stabilizers include those that can stimulate economic growth so as to get that extra needed public income for the debt sustainability. Therefore the paper proposes to use an innovative tool for the science of public finance, the Kalman filters(2). They will support the public finance manager (authorizing officer) to acknowledge what will the the future trend for the recorded current income in future periods of time, because on this basis he can predict those situations with the risk of default that may threaten the public debt service. Furthermore, in order to exercise a pudent control over the risk of default(3), the financial manager can anticipate situations that will appeal to refinance public debt, caused by the effect of volatile evolution on the current revenue received from the budget.

II. State of knowledge for the use of Kalman filters in economics

R.E. Kalman published his famous work, in the early '60s, which describes a recursive solution to the discrete data linear filtering problem(4). Since then, largely due to advances in digital computing, Kalman filter has been the subject to extensive research and applications, especially in autonomous or assisted navigation. Among the first approaches to the filter idea, as an introduction to the general idea of Kalman filter, can be found in Chapter 1 of Peter Maybeck's - Stochastic Models, Estimation and Control(5), while a more complete introductory discussion can be found in Harold Sorenson's books(6). More extensive references are included in the work of Gelb(7), Grewal, Lewis Jacobs and Evensen(8).
Prior to presenting the state of knowledge, for the Kalman filters, it is necessary to analyze the work of Norbert Wiener(9). Wiener described an optimal response for a finite impulse (FIR) filter, meaning the average quadratic error. We will not discuss his solution here, even though there are many parts in common with Kalman filter. Just remember that his solution uses both auto correlation and cross correlation of the signal received with the original data to obtain a response in a filter impulse.
Kalman has also submitted a option for quadratic errors of the filter. However Kalman's version has some advantages over Wiener's study: avoids the need to determine the filter impulse response. Kalman described his procedure of filtration using state space techniques, which unlike Wiener's method, allows a filter to work as an adjustant, as a filter or as a predictor. The latter depending on the applicability of the filter, Kalman filter that is able to be used to predict future data proved to be a very useful feature. This has led to the Kalman filter to a wide range of tracking and navigation usage. Defining the filter in terms of state space methods also simplifies the implementation of the filter in discrete values, another reason for its widespread use.
The concept of debt sustainability has been discussed for at least two decades. Two general approaches have been followed, first considers that the interest rate at which a government can not borrow more than the rate of economic growth, so that the ratio of debt to GDP does not increase, and an unsustainable debt does not appear. Another approach considers that if there is a constraint on the present loan, which could limit the amounts borrowed, then it would be the main criterion for achieving sustainability. Gupta (1992)(10) in an exponential work, consider

these two approaches to Asian countries. In both approaches, it underlines and show the importance of two key issues: the selection of an appropriate interest rate and use taxes. A useful discussion on these aspects and positions of different authors is presented in his book (Gupta, 1992).

III. The arithmetic of the public debt and of the budget deficit

Public debt sustainability should be a major concern of central governments to avoid default risk and a deterioration of the country's credit rating may lead to unjustified price rises in the cost of procuring the necessary financial resources to support any state deficits budget.(11)

Any theory of public debt should be confined to a seemingly simple question but with major implications for the public budget: How to ensure sustainability of public debt?

To answer this question it is necessary to resort to science expertise in public finance. This implies that government revenue should be divided into two distinct categories, namely current income, respectively capital income. In turn, public spending must also be structured into two categories, namely current expenditure, respectively capital expenditure. It thus forming two major categories of budgets for public debt sustainability analysis, namely the operational budget, respectively capital budget.

The operating budget is suposed to sustain the current activity of the state to finance the operating costs of the public institutions whose financing comes from the public budget, while the capital budget expenditure may support the investments and public expenditures on interest and on loans. The two budgets, apparently unconnected, communicate via the operational budget balance.

In the current income category, are included according to the speciality theory, tax and non-tax revenues consisting primarily of taxes and/or revenues from concessions and rentals, while operating expenses category includes personnel costs and material expenses and services are necessary for the public institutions financed from public funds.

Denoting the category of the state's current revenue with (V_C) and current project expenditures with (Ch_C), results that the first condition of public debt is the result of budget implementation for the operational budget to record levels to be higher or at least equal to debt service (S_{DP}), according to the relationship:

$$V_C - Ch_C = S_{DP} \quad (1)$$

Taking into account the relationship (1) and the analytic form of the income and expenses above the current relationship can be rewritten as follows:

$$V_f + V_{nef} - (Ch_{pers} + Ch_{mat} + A_{chmat}) = S_{DP} \quad (2)$$

In the literature the difference between current revenues and current expenditures is known as gross operating results which may take the form of surplus / deficit. If from the gross operating result is deducted the debt service we achieve the net operating result .

Through the arithmetic describing the public debt sustainability can be easily deduced at least two particular cases obtained by comparing the specific sustainability of public debt to the gross operating result, namely:

Situation 1 $\frac{S_{DP}}{R_{bop}} \geq 1$ corresponding to a situation where debt is sustainable from the gross operating result which implies that there isn't a risk of entry of default on debt service. However the situation is a bit risky if the value of the indicator is close to one, which can trigger a long-term warning for the central government, on the ability to repay the debt service.

Situation 2 $\frac{S_{DP}}{R_{bop}} \prec 1$ is corresponding to a situation in which the debt service is not sustainable from the gross operating result which means that central governments should support the refinancing of public debt service, through borrowing money from the public, which often can be quite expensive because of the high cost of the financing sources, necessary to cover the debt service.

Whatever values the indicator may take is important to note that the literature is purely innovative. Although the service is calculated as the ratio between the debt service level (S_{DP}) and gross operating result (R_{bop}) known as the indicator of debt sustainability (I_{SDP}), is the "supplier" of information about public debt service related to financial capacity that the state has to support for the debt service on which committed.

It is interesting to consider in this context, central government policies that can promote to ensure the sustainability of public debt service. There are two categories in my view that government policies can promote.

The first type of fiscal policy-oriented to public debt sustainability refers to the policy of increasing the income fiscal tax base. That is why one of the indicators examined in this context is the ratio of debt service and gross domestic product (GDP) to identify the possible growth potential at national level but also additional public budget revenues. It should be noted that there is no alternative to increase tax rate for tax revenue but this kind of fiscal policy is rather difficult to implement because of the opositiona tha usually the taxpayers manifest.

The second type of fiscal policy also focused on the public debt sustainability refers to rationalizing public spending. This policy may be a rational only if applied on the operating costs of public institutions. Otherwise, the damage brought to capital expenditures may inhibit the incentive effect of the public debt needed for economic growth especially in times of economic crisis.

Also important is the correlation between the budget deficits and public debt, because any explanation that we try to give for public debt in terms of substantiating its necessity, it is based on the deficit recorded during the financial year.

Between two consecutive periods of time, the deficit is determined by the level of government spending (Ch_G), the level of public revenues collected on the budget (V_{BUG}) and the interest expense due to debt service set according to interest rate (i) and public debt (D_p) as a relationship form $(i \times D_t)$.

Arithmetic equation describing the budget deficit will be given by:

$$D_t - D_{t-1} = Ch_G - V_{BUG} + i \times D_t \quad (3)$$

Analysis of this equation of evolution of the budget deficit without going into a detailed analysis that otherwise has been made in the literature, shows that every positive (ΔD_t) has to be covered by debt service, while negative values of this indicator can lead to identify the implications of budgetary surpluses already known to specialists in the field.

Regardless of the correlation that must exist between the budgetary deficit and public debt service is worth noting that for a manager of public financing resources, is important the knowledge of future evolutions, with a certain degree of certitude, for the operating income and the current expenditures. This is because it offers the possibility to anticipate future risks related to failure of payment of central government and can be taken to avoid such risk situations that may endanger the national security of states.

The response for all this information for the financial manager will be given by using an innovative tool in the science of public finances, namely Kalman filters.

IV. Kalman Filter mechanism

Kalman filter addresses the general problem of trying to estimate the $X \in R^n$ state for a discrete time process, described by the following linear stochastic equation:

$$x_{k+1} = F \times x_k + w_k; \text{ (3)}$$

where x_k a state vector of the process at the moment k, F is the transition matrix from the state k to the state k+1, and is supossed to be stationary for the whole period, and w_k is the white noise asociated to the process the known covariance, Q.

The observation upon this variable can be modeled in the following form:

$$z_k = \mathrm{H} \times x_k + v_k \text{ (4)}$$

where z_k is the measurement of x at the moment k; H the connection betweend the state vector and the measurement vector, and is supposed to be stationary during the whole period, while v_k is considered to be a measurement error, a white noie,(12) with the known covariance, R.

The differecent between the estimated state $\hat{x}_k$ and x_k is also known as error

$$f(e_k) = f(x_k - \hat{x}_k) \text{ (5).}$$

Considering the error function that is strictly positive and monotonically increasing, we can write $f(e_k) = (x_k - \hat{x}_k)^2$. Since it is necessary to consider the filter's ability to forecast data series over long periods of time, we rewrite the loss function E($f(e_k)$), from where it results the least square function: $e_t = E({e_k}^2)$.

Consider that the two covariances for the errors are: Q=E[$w_k, {w_k}^T$] and R=E[$v_k, {v_k}^T$].

The least square function is P_k=E[$e_k, {e_k}^T$], where P_k is the covariance matrix at the k moment. This equation can be extended to the following form:

$$P_k\text{=E}[(x_k - \hat{x}_k)\,(x_k - \hat{x}_k)^T]. \text{ (6)}$$

In demonstrating the Kalman filter equation, start by putting as objective finding an equation that uses an a posteriori state estimate $\hat{x}_k$ with an a priori estimation $\hat{x}_k{}'$, and the difference between the measured value z_k and a prediction of the measure, $\mathrm{H}\hat{x}_k{}'$,

$$\hat{x}_k = \hat{x}_k{}' + K_k\left(z_k - \mathrm{H}\hat{x}_k{}'\right) \text{ , (7)}$$

where K_k is considered the Kalman gain. This is a matrix that has the role to minimize the a posteriori covariance error.

$$\text{Note } i_k = z_k - \mathrm{H}\hat{x}_k{}'. \text{ (8)}$$

By making replacements in the covariance matrix P_k , the derivation of the function by K_k, and the utilization of the matrix trace we obtain the following formula for K_k:

$$K_k = P'_k H^T (H P'_k H^T + R)^{-1}. \text{ (9) (13)}$$

The covariance matrix for i_k prior denoted is $HP'_kH^T + R$.

The projection of the x_k variable, is $\hat{x}_{k+1}{}' = F\hat{x}_k$.

To complete the repetition is necessary to find an equation that projects the error covariance matrix in the next moment, k + 1. This is accomplished by writing an expression for the previous error:$e'_{k+1} = x_k - \hat{x}_k{}'$

$$= (\mathrm{F}\, x_k + w_k) - F\hat{x}_k{}' =$$
$$= \mathrm{F}e_k + \mathrm{w}_k \quad \text{(10)}$$

Using equation 4 and rewriting it for the moment of time k, we get:

$$P'_k\text{=E}[(\mathrm{F}e_k + \mathrm{w}_k), (\mathrm{F}e_k + \mathrm{w}_k)^T] \text{ (11).}$$

Given that between w_k and e_k, there is any correlation beceause w noise accumulates between moments k and k +1, while the error is that by the time k. Therefore:

$$P'_k\text{=E}[e'_{k+1}e^{T\prime}_{k+1}\,]=$$

$$=\mathrm{E}[Fe_k(Fe_k)^T]+\mathrm{E}[w_k w_k^T]=$$
$$=FP_kF^T+Q \quad (12)$$

In essence the use of Kalman filter requires the use of two sets of equations:

For prediction

-The prediction of the future state

$$\hat{x}_{k+1}=F\times\hat{x}_k+w_k$$

-The prediction of the covariance error

$$P_k'=FP_kF^T+Q$$

For measurement or correction,

-The Kalman gain calculus

$$K_k=P_k'H^T(HP_k'H^T+R)^{-1}$$

-The update of the estimate on the extent of z_k

$$\hat{x}_k=\hat{x}_k{}'+K_k\left(z_k-\mathrm{H}\hat{x}_k{}'\right)$$

-The update of the covariance eror for the next step

$$P_k=(I-K_kH)P_k'. \quad (14)$$

V. Kalman filters use the debt sustainability analysis

In our analysis, we developed a Matlab (15) code for the program, code for mathematical equations proposed by Kalman filter, which, based on the analyzed data sets, generated estimates for periods of time so that we had real data sets comparison estimates, but also on future periods of time, for the purposes of prediction.

Following repeated simulations, needed to allocate the value of the transition matrix F, measurement matrix H, the two white noises, v and w, and their covariances, Q and R, the best simulations were obtained by assigning the following values :

$F=\begin{bmatrix}0&1&0&dt&0&0\\0&0&1&0&dt&0\\0&0&0&1&0&dt\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$; Where dt=1. $H=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\end{bmatrix}$, and matrixes Q and R where identity matrixes of order 6 and 2. (16)

The key element in the ecuational system is the Kalman gain, which is the variable that stores the past evolution of the data series. This projected increase accuracy of the information as the number of variables taken into account in constructing the time series is more widespread.

In Fig. 1 is shown the evolution of Romania's foreign public debt service for a period of 74 months December 2004 - January 2011. Using the Kalman filter, we generated starting with the 40th month, predictions for the public debt service evolution, comparing the data obtained in parallel with the real values of the debt service recorded. One can see a aproximation for the projected Kalman filter values. The values are predicted for a single month, then renewed their series filter data used to generate the next prediction.

- The real recorded value for public debt
- The value predicted through the Kalman Filter
- The real recorded trend

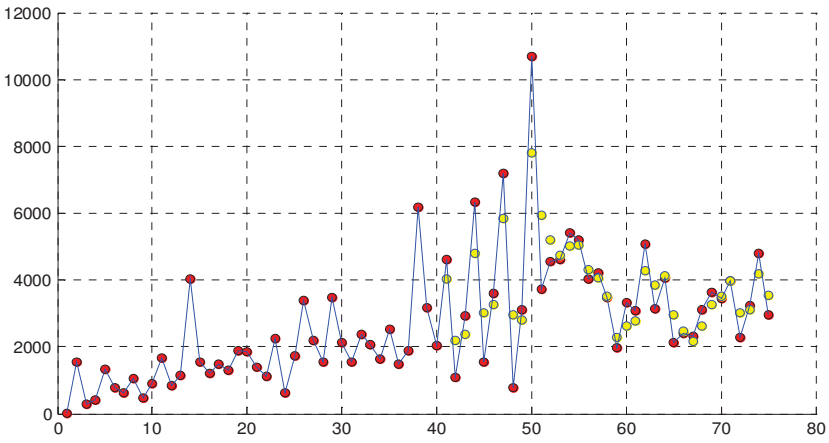

Fig 1. The evolution of the total external debt service and the prediction through the Kalman filters, comparison for the last 25 values of the data series; Monthly data series december 2004 – january 2011

Date source: bnro.ro, insse.ro

In order to achieve public debt sustainability at the national level is essential to have a number of indicators for the present and past values, so you can make accurate estimates of the future development. Thus, using Kalman filter, for the time series for Romania's foreign debt between January 2005 - January 2011, we generated future values for external debt over a period of 10 months. One can see that will continue the future upward development trend, but not with the same intensity.

In figure 3, one can observe the errors or deviations between the expected values generated and the actual values. You can also see the main feature of the use of Kalman filter, namely, the generation of more precise data on the growth of the data series used. Thus, the Kalman filter is not a simple prediction mechanism, it is a mathematical tool, generating an active process of updating the information in order to generate more reliable information.

However, in our research to assess the current and future sustainability of external debt, to have a clearer picture, we need to consider as we mentioned in the paper, the financial capacity of the budget to cover the public debt service.

Figures 4 and 5 show the evolution for the relevant indicators with a view to public debt sustainability analysis. A future perspective on the level of the revenue for the state budget, may provide an important advantage in view of future risk reduction, given by an unsustainable level of debt. Based on the Kalman filter used in fig. 4 can see a negative forecast for the developments in the next six months for the operational costs, while income level is only slight fluctuations over the prospect of the final level of expenditure growth.

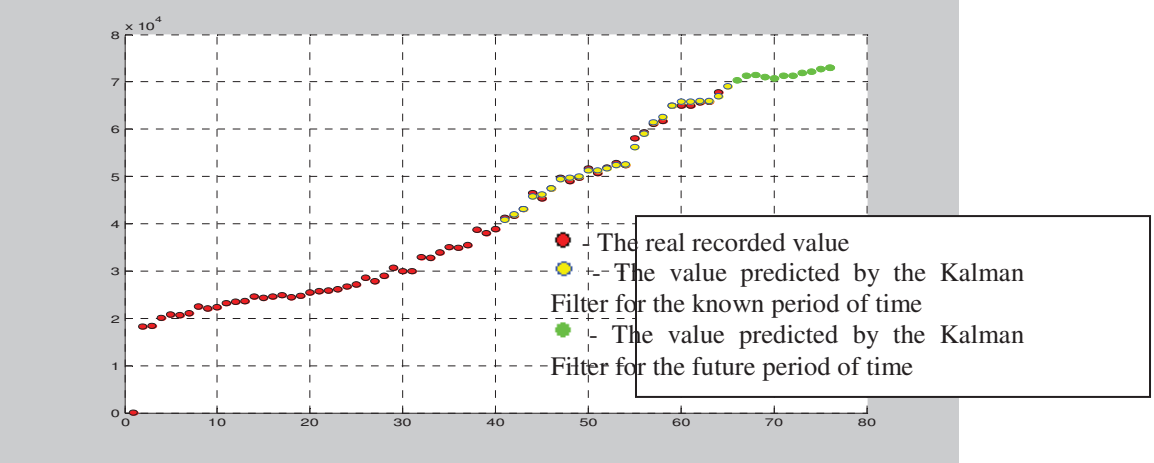

Fig 2. Evolutia datoriei externe, si previziunea prin filtrele Kalman
Serii de date lunare ianuarie 2005 – ianuarie 2011 (mil. E)
Sursa date: bnro.ro, insse.ro

As mentioned above, a level of debt service to be covered by a lower level of the primary deficit, or preferably, to a surplus. In Figure 5 we can see an upward trend in the gap between deficit and debt service. Beyond the 25th month, is found as the difference between the two indicators, create the base of a costly debt service. Expected future developments based on Kalman filters, also reveal a negative trend, especially in the penultimate month, when this difference tends to reach one of the highest levels.

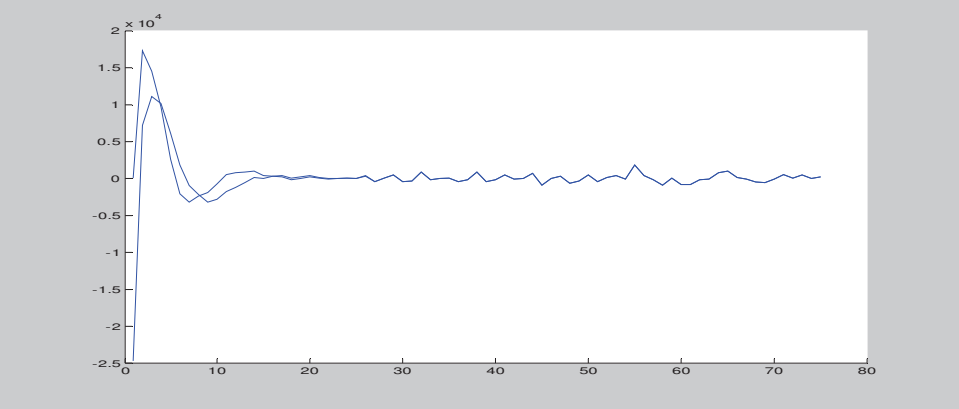

Fig 3. The evolution of the errors obtained by applying the Kalman filter on the public external debt and the real recorded values; MATLAB 7.9

VI. Conclusions, proposals and future research

Public debt sustainability is an issue of major concern for central and local governments in the European Union. Whatever indicators are analyzed to study the sustainability of public debt it remains as a fact that public funding of any government debt service can not be achieved only with a positive result of the implementation of the operational budget. The results of the operational budget should be sufficient to cover the debt service. Based on these assumptions we have developed a relevant indicator, that can provide factual information about public debt sustainability. This indicator was calculated as the ratio between debt service and operating budget outcome. Positive values indicate a debt sustainability while below par values indicate the contrary that may affect thepublic debt service,respectively the risk of default.

For any manager of financial resources is important to know information about the predictability of the operationat income and operational expenditures. How predictable are they? Which are the future periods of time that may affect the sustainability of the public debt? It is a question which we attempted to answer through the application of the Kalman Filter.
It was found after the application of the Kalman Filter, that the operational income recorded a fairly stable evolution, while operational expenses for the period recorded a high enough fluctuation.
Debt service, during the projected period through the Kalmanfilters, also has a fairly stable level, continuing the late of trend. This means that debt service can be affected by the results of the operational budget deficits. That is why careful monitoring is necessary for the operational expenses of public institutions to reduce the default risk for the public debt service payment.
The advantages of using the Kalman Filter to predict, from classical methods, is given by the recursive mechanism, effective and accurate directly proportional to the series features the processed data.
Our future research directions in this area will move towards efficient fine tuning of the transition matrix and white noise used in the mathematical model. The study will also cover deepen regarding the progress of non-stationary series, where the results of Kalman Filters are known in the literature as being superior to other forecasting methods.

VII.Notes

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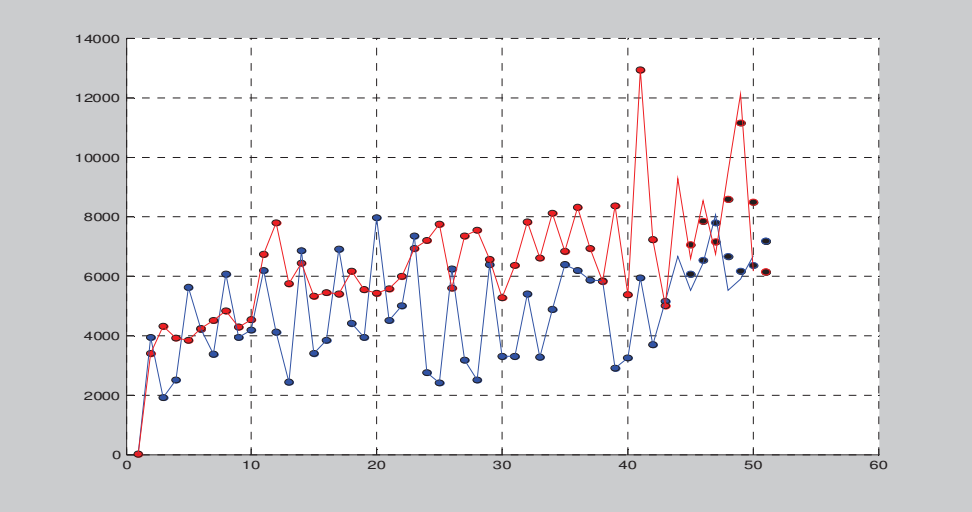
ANEXES



- The value recorded for operational expenditures
- The value recorded for operational revenues
- The value by the Kalman Filter for the future period(operational expenditures)
- The value by the Kalman Filter for the future period(operational revenues)

Fig 4. The evolution of the operational revenues and expenditures for the state budget (march2007-december 2010) and the prediction for 6 months (mil. RON)

Source of data: insse.ro, mfinante.ro, MATLAB 7.9

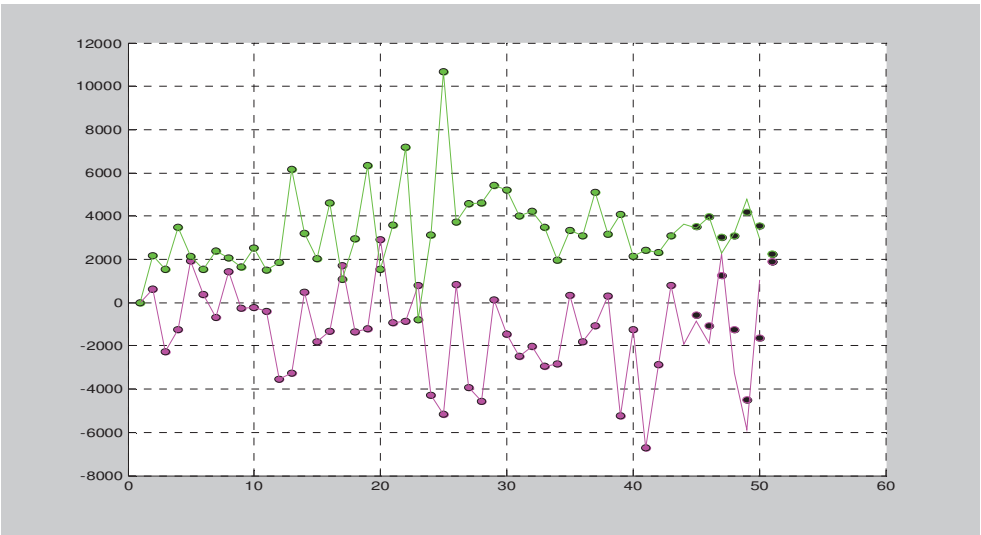


Fig 5. The evolution of the public debt service and the primary budget deficit (march2007-december2010) and the prediction for 6 months

Source: insse.ro, mfinante.ro, MATLAB 7.9

- The value recorded for the public debt service
- The value recorded for the operational primary budget deficit
- The value predicted by the Kalman Filter (public debt service)
- The value predicted by the Kalman Filter (operational primary budget deficit)