CORPORATE VALUATION USING TWO-DIMENSIONAL MONTE CARLO SIMULATION

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In this paper, we have presented a corporate valuation model. The model combine several valuation methods in order to get more accurate results. To determine the corporate asset value we have used the Gordon-like two-stage asset valuation model based on the calculation of the free cash flow to the firm. We have used the free cash flow to the firm to determine the corporate market value, which was calculated with use of the Black-Scholes option pricing model in frame of the two-dimensional Monte Carlo simulation method. The combined model and the use of the two-dimensional simulation model provides a better opportunity for the corporate value estimation.

Keywords corporate valuation, free cash flow to the firm, real options, Black-Scholes option pricing model, two-dimensional Monte Carlo simulation, R statistics
JEL code: G32, C15

1. Corporate valuation

There is an ever-increasing need for financial valuation services pertaining to nonpublic companies and subsidiaries, divisions, or segments of public companies. Nowadays, the main purpose of the companies to increase their corporate value. In order to measure the value creation of a company, namely the increase in value compared to previous one, it is necessary that we can properly estimate the company's current value. The new reality presents a challenge to business managers: the need to manage value and to focus as never before on the value their corporate-level strategies are creating. Copeland-Koller-Murrin (2000) write in their book: „*We believe that clear thinking about valuation and skill in using valuation to guide business decisions are prerequisites for success in today's competitive environment. … Underlying it is our basic belief that managers who focus on building shareholder value will create healthier companies than those who do not. We also think that healthier companies will, in turn, lead to stronger economies, higher living standards, and more career and business opportunities for individuals.*"
The valuation plays a key role in many areas of finance - in corporate finance, mergers and acquisitions and portfolio management. In general, there are three approaches to valuation. The first, discounted cashflow valuation, relates the value of an asset to the present value of expected future cashflows on that asset. The second, relative valuation, estimates the value of an asset by looking at the pricing of 'comparable' assets relative to a common variable such as earnings, cashflows, book value or sales. The third, contingent claim valuation, uses option pricing models to measure the value of assets that share option characteristics. (*Damodaran, 2002*)
To build our company valuation model we have used a kind of the combination of the methods above-described by Damodaran. The combined methods were designed to obtaining more accurate results and to taking into account the forecast uncertainty during the calculations. Still combining of the methods are needing, because the company valuation is a complicated process.

As business structures become more complicated, business valuations become more complicated. As transparency in corporate financial reporting becomes more desirable and less attainable, business valuations become more complicated. And, as securities market cycles become more exaggerated and less predictable, business valuations become more complicated. (Reilly-Schweihs, 2004)
The corporate valuation is also important to measure the performance of the management. Value creation is the ultimate measure of performance for a management team. (Copeland-Koller-Murrin, 2000)

2. The corporate valuation model

There are only three approaches to value any asset, business or business interest:

1. the income approach;
2. the market approach;
3. the asset approach.

There are no other approaches to value. However, there are numerous methods within each one of the approaches that the analyst may consider in performing a valuation. All three approaches should be considered in each valuation. However, it is not common to use all three approaches in each valuation. (Hitchner, 2003)
To determine the market value of the company we have developed a combined model whose main stages are presented in the ***Figure 1***. On behalf of the more efficient estimation we had combined more known methods and the corporate value as the final result was calculated using the so-called two dimensional Monte Carlo methods with utilization of intermediate results of the specific methods. The raw data necessary to the modeling was collected from the financial statements some of the companies listed on the Budapest Stock Exchange. We chose companies belonging to the stock exchange with the intention to compare the calculated results with the stock market capitalization of the firms.
The value determination was carried out for five companies. However, due to size limitations of the article the details of the calculation are only presented for one company (*Danubius*).

2.1. Calculation of free cash flow to the firm

To preparation of forecasting which is necessary to determine the asset value of a company we have used the last five years data (2004-2008) of the firms downloaded from the Budapest Stock Exchange web-page. The calculations have been performed in Microsoft Excel 2007 and the R statistical system. R statistical system is an integrated suit for software facilities for data manipulation, calculation and graphical display. R is an environment within which many classical and modern statistical techniques have been implemented. (Venables-Smith, 2009) The commands of R can be executed from the Microsoft Excel, and we have used this possibility.
During the forecasting we have calculated those values which were necessary to determine the free cash flow to the firm (FCFF). We used the company's growth ratio to make the forecast of the data. The forecast was based on the company growth rate[528] and marginal taxation rate[529]. To generate the latter rates we have used the normal distribution with parameters based on the averages and standard deviations of the previous years' data. (***Table 1***)
The forecast results are in the ***Table 1***. The FCFF values were calculated using the after-tax operating profit, as follows

$$FCFF = \text{After-tax operating profit} + \text{Depreciation} - \text{Investments} - \text{Changing in non-cash working capital} \quad (1)$$

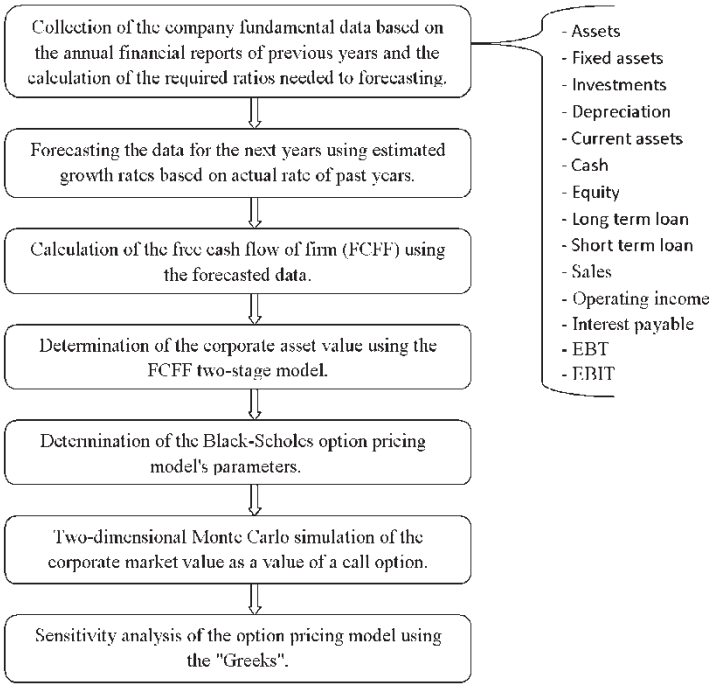
[528] mean=9.14, sd=2.23
[529] mean=22.55, sd=6.93

In case of Formula 1, the cash flow does not include the benefits arising from the interest payments, since the tax-saving effects of the credits has been taken into consideration in case of the determination of the cost of capital.
The corporate asset value was calculated by using of the Gordon-like two-stage valuation model, in consideration of the following relations:

$$V_A = \sum_{t=1}^{n} \frac{FCFF_t}{(1+r)^t} + \frac{P_n}{(1+r)^n} \quad (2)$$

$$P_n = \frac{FCFF_n * (1+g)}{(r-g)} \quad (3)$$

Figure 1: The process and the raw data of the corporate valuation



The growth rate (g) used to the determination of the terminal value (P_n) was calculated as an average of the forecasted growth rates (*the growth rate from 6th year* - Table 1). (Formula 3) The weighted average cost of capital (r – *required rate of return* – Table 1) was calculated as an average of the yearly weighted average costs of capitals. (Formula 2, 3)

2.2. Real options in corporate valuation

The calculation is essentially based on the assumption that the market capitalization of a company determines the market value of a joint stock company. Two essential factors play an important role in general to elaborate the market price: the actual performance of the company and the investor expectations. The riskiness of a company reflects in the volatility of the stock prices.
The new economy provides a challenge for the corporate decision-maker. Corporate valuation may no longer depends on traditional fundamentals but rather on future expectations. Investment strategies with high risks and uncertainty or irreversible corporate decisions coupled with managerial flexibility provide the best candidates for real options. Real options is a systematic approach and integrated solution using financial theory, economic analysis, management science, decision sciences, statistics and econometric modeling in applying options theory in valuing real

physical assets, as opposed to financial assets, in a dynamic and uncertain business environment. (*Mun, 2002*)

To determine the corporate market value we have used the Black-Scholes options pricing model. The value of a real options depends on five basic variables:

1. the value of underlying risky asset (**S**) – ***corporate asset value***;
2. the exercise price (**K**) – ***face value of the outstanding loans***;
3. the time to expiration of the option (**t**) – ***weighted average maturity of loans***;
4. the standard deviation of the value of the undelying risky asset (**σ**) – **variance of the firm's share**;
5. the risk-free rate of interest over the life of the option(**r**) - **treasury bond yield rate adjusted to the option lifetime**. (*Copeland-Antikarov, 2003*)

Table 1: The corporate asset value calculation based on forecasted data

million HUF

Titles of data	Forecasted years				
	1	2	3	4	5
Growth rate	10,57%	8,49%	9,44%	8,22%	7,42%
Assets	91 363	99 124	108 482	117 398	126 111
Sales	49 417	53 615	58 677	63 499	68 212
Operating profit	2 324	2 521	2 759	2 986	3 208
Marginal taxation rate	32,35%	18,82%	29,02%	11,66%	28,10%
After-tax operating profit	1 572	2 047	1 958	2 638	2 306
Invested assets	83 767	90 882	99 462	107 636	115 626
Depreciation	4 937	5 357	5 863	6 344	6 815
Investments	4 685	5 083	5 563	6 020	6 467
Changing in non-cash working capital	274	297	325	351	378
Free Cash Flow to the Firm	1 551	2 023	1 933	2 610	2 277
Terminal price					***38 625***
Present value of FCFF	*1 340*	*1 512*	*1 248*	*1 457*	*1 098*
Corporate asset value	**45 279**				
growth rate from the 6. year	*9,26%*				
required rate of return	*15,70%*				

The weighted average maturity of loans was estimated with the using of the notes of the accounts of corporate annual reports. The variance of the firm's shares was calculated with the using the closing price of the company's shares what was downloaded from the web-page of the Budapest Stock Exchange. (Table 2)

To calculate the corporate value we have used the following formulas of the Black-Scholes model:

$$\text{value of the call option} = S * N(d_1) - K * e^{-rt} * N(d_2) \quad (4)$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) * t}{\sigma * \sqrt{t}} \qquad d_2 = d_1 - \sigma * \sqrt{t} \quad (5)$$

2.3. Two dimensional Monte Carlo simulation

A two-dimensional (or second-order) Monte Carlo simulation is useful to estimate the "uncertainty" in the risk estimates stemming from parameter uncertainty. A two-dimensional Monte-Carlo simulation is a Monte-Carlo simulation where the distributions reflecting "variability" and the distributions representing "uncertainty" are sampled separately in the simulation, so that "variability" and "uncertainty" in the output may be estimated separately.

The developers of the two dimensional simulation model write in their modell application guide (Pouillot, R. et al., 2010) the followings: „*a QRA* (Quantitative Risk Assessment) *should reflect the „**variability**" in the risk and calculate the"uncertainty"associated with the risk estimate. The"variability"represents temporal, geographical and/or individual heterogeneity of the risk for a given population. The "**uncertainty**"is understood as stemming from a lack of perfect knowledge about the QRA model structure and associated parameters.*" On basis of Cullen and Frey book (1999, p.3.), what can be regarded as the theoratical background of two dimensional simulation, the previous definition is supplemented by the followings: „*In general, variability can not be reduced by additional study and measurement. Random and systematic measurement errors as well as reliance on models and surrogate indicators, all sources of uncertainty. … In confronting variability and uncertainty a decision-maker stands to better understand the degree of variance int he full distribution of exposure or risk, the impact of various assumptions, data gaps, and model choice on decision making, and the most fruitful avenues for further study."*
The input parameters of the simulation model are presented in Table 2.

Table 2: The input value of the simulation model

Corporate asset value		45 279	million forint
Average maturity of loans		5,54	év
Variance of the corporate shares	**U**		
minimum		8,85%	
mean		9,78%	
maximum		10,70%	
Outstanding loans	**V**		
minimum		21 310	million forint
mean		23 966	million forint
maximum		27 511	million forint
Five-year treasury bond yield	**U**		
minimum		4,50%	
mean		6,00%	
maximum		8,50%	

To estimate the stochastic variables marked U (*uncertainty*) and V (*variability*) letters we have used the triangular distribution with the parameter values in Table 2. (Kotz-van Dorp, 2004)

3. Results of simulation

To execute the simulation model we have used the 'mc2d'[530] package of R statistical system.
The simulation statistical results are presented in Table 3. The statistical indicators of Table 3 show that the value of the coefficient of variation is very low, and the interquartile range[531] is relatively small, which show that the estimated values are located in a relatively narrow interval around the mean and median around. All these can be easily seen in the boxplot diagram.
To determine a confidence interval for the mean and median the program was used the one sample t-test and the Wilcoxon signed rank test with continuity correction. The calculation results are shown the Table 4. We can see that the intervals have a very small scale.
To measure how the value of an option will change when one of the input variables changes while the others remain the same, we have used the „Greeks" (*delta=0.7021, gamma=6.86*10^{-7}*).

Table 3: The statistical results of the simulation

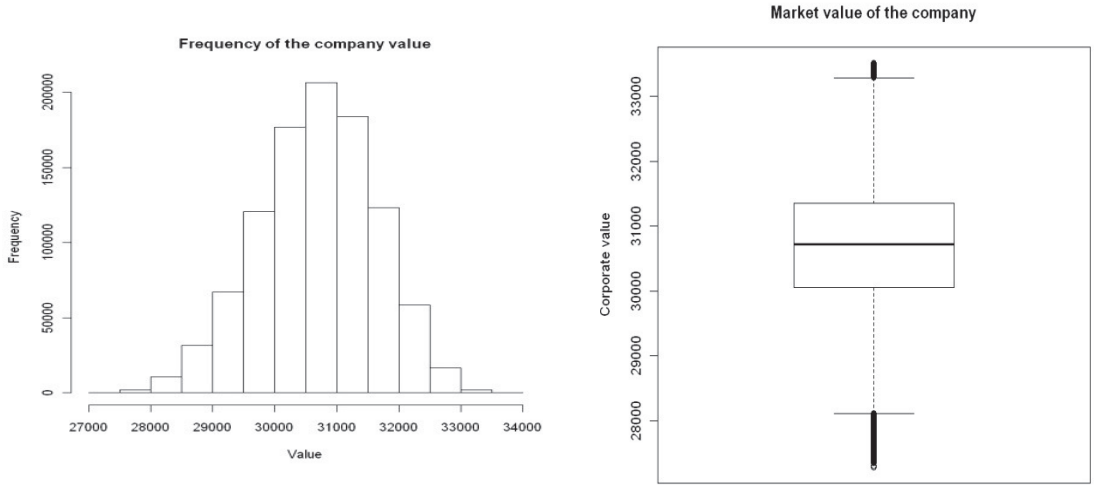
[530] Monte-Carlo à Deux Dimensions
[531] interquartile range = 3rd quartile – 1st quartile

Statistical indicators	Minimum	1st quartile	Median	Mean	3rd quartile	Maximum	Standard deviation	Coeff. of variation	Inter-quartile range	Kurtosis	Skewness
Value of the company	27 290	30 060	30 720	30 680	31 350	33 510	939	3,06%	1 290	-0,2228	-0,1972

Table 4: Confidence interval estimation for mean and median

Statistical indicators	confidence interval (95%)		estimated value
	lower bound	upper bound	
mean	30 683	30 687	30 685
median	30 700	30 704	30 702

Figure 3: The diagram results of simulation[532]



4. Conclusions

The performed calculations show that the constructed model can be applied in determining the market value of companies. The stock market capitalization of the company's shares was HUF 29,330 at the end of 2008, which is 2.49% less than the estimated average value. The model slightly over estimates the current market value.

The sensitivity tests carried out have confirmed the applicability of real options.

Probably we get better results of the corporate value if we would have a more accurate internal company information. We have obtained similar values in case of the other analyzed companies too. In each case, the estimate is well approximated in the current market value.

We believe that the presented model is useful to determine the value of the companies whose shares are traded on the stock exchange.

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[532] The result variable contains 1 million piece of the estimated value.

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