

# THE BERTRAND MODEL OF THE SINGLE MARKET

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*Starting with the signification of the rationality hypothesis when the agent's contentment is directly affected by the other agents' decisions, the theory of games defines solutions for solving different situations of conflict. The economic actors have different behaviours of the Single Market. Oligopoly strategic behaviours were analysed by the Bertrand model. The two types revealed in the work show that strategic interactions are sensitive to the companies' features, products and markets. Regarding the situation when we have an oligopoly competition, the companies make interdependent decisions in the environment affected by risk and uncertainty of the Single Market. For this reason it is an opportunity to study the structure of oligopoly type of the Single Market with the aid of non – cooperative games.*

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## **1. Introduction**

The economic models and the practice of economical - mathematics constituted an excellent instrument for studying the economic games, stimulating the research in this field. In the last decade, a series of methods regarding the representation of the economic theory were used in order to study the evolution of state parameters of the social- economic field. The category of systems highly studied in the economic dynamism are those who pattern the business circle, patterns of economic increase and patterns which study the costs game in a dynamic perspective. In the last decade, from time to time, evolution and chaotic behaviour were noticed in the economic patterns. The main conception of the economic science was demonetized, which says that the economic balances are constant even with the lack of external shocks, the economy leaning towards a stationary state.

From this point of view, conducting a study in a dynamic environment, about costs mechanism seems to be a very important problem. Taking into account the cost which is an economic phenomenon, this work trying to approach it, using a modern instrument of work, belonging exclusively to mathematics, which is *the game theory*.

We are interested in the theoretical results which are revealed in literature regarding the theory of games starting with the key concepts of this one: games, strategies, balance, game value, etc. the research is focused on the main oligopoly market structures from the microeconomics point of view. The theory of games studies the human behaviour in situations of conflict, where the reason is contrary to the reason, each of the parties involved being able to analyse and to decide in order to reach their own targets. It emphasises the meaning of the rationality hypothesis when the contentment of a person is directly affected by the other agents' decisions and defines solutions for various situations of conflict. The theory of games is a method of research for strategic interaction situations, where the economic agents are aware of the interdependence which exists between them and each of them will make their own decisions taking into account the others' behaviour.

*The theory of games was first mentioned and historically related to the year 1944 when the mathematician John von Neumann and the economist Oskar Morgenstern publish the famous work: Theory of Games and Economic Behaviour*

Augustin Cournot studied in 1838 the operation of the oligopoly markets where each company takes action knowing that its volume of production affects the market cost. In 1833 J. Bertrand studied the operation of the oligopoly markets where the companies with constant efficiencies produce the same product settling the selling price. The result mentioned by Bertrand is known as Bertrand paradox<sup>186</sup>. In 1934 Stackelberg shows that some companies can be leader and that are able to impose the price to the others. The leader company, as a barometer company, knows best the market situation and has the means necessary in order to control the counter party. This doesn't mean that the company is the most powerful but well informed and organised.

The following question comes up: Which of the behaviours mentioned above should be followed of the Single Market? In order to answer this question, a theory was needed which could explain the interactions between companies. This is the great contribution of the game theory. It allows the elaboration of an analytical framework regarding the situations when an agent's decisions can affect the earnings of the other agents.

The theory of games studies the manner in which the rational decision are made by people in interactive situations when the results of their action depend directly on the others; actions. In this case, where the coordination of the individual actions is adjusted by competition, each economic agent (named player in the game theory) must forecast the future actions of the other agents and then optimise their own behaviour depending on the results. In the last two decades the rent-seeking type of dynamic games were analysed (to share the fortune). A systematic study of the balance point for this kind of games was made by Okuguchi K. and Szidarovsky F. who revealed that these games are equal to the Cournot type of oligopoly with price hyperbolic function.

The theory of games hyphens the meaning of the rationality hypothesis when the contentment of the person is directly affected by the decisions of the other agents and defines solutions for different situations of conflict. Out of this reason knowing the analysis instruments of this theory is essential nowadays, the theory of games constituting a real matrix of the contemporary financial theory. Its postulates are based on the analysis of the people's interdependent relations.

The economic description and analysis must be oriented towards solving the conflicts caused by the problems of redistribution. The approach moves from solving problems related to assignment towards the analysis of the availability restrictions which influences the resources assignment and distribution. In conclusion, the economic reality can't be properly presented by a static approach, as the traditional microeconomics theory does it, but it must be seen as a process, with the help of a dynamic approach, from the new microeconomics perspective.

## ***2. Competitive strategies of the Single Market.***

Some of the economic agents have a different behaviour on the market affecting the other economic agents' behaviour. This type of reality resides in the "market structure" concept. The meaning of *market structure* represents the features of a market by the number and the relative power of the companies which operate on the market having the purpose of settling their behaviour and its consequences on the economic efficiency of the economic system. The contributions of the game theory to the study of the Single Market can solve some problems related to the costs system and to the intelligence exchange. Competition was and is related to the behaviour hypothesis of the economic agents and to the relative hypothesis of the market operation.

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<sup>186</sup> Binmore K., *Jeux et théorie des jeux* Ed. De Boeck Université, Bruxelles, 2001, p.23

The market structures vary depending on: the influence on the costs settlement; the companies' production of standardized or non - standardized products; the companies' possibility to enter on the market; the publicity, the products' features, etc. The oligopoly means a market structure controlled by a restricted number of producers, the actions of each producer affecting the others competitors' actions. If a producer cuts the price in order to increase the sales, then its competitors will react by cutting their price too, thing which will determine a profit decrease for the first company. Before making the decision regarding the price cut, the oligopoly company would have to analyse first the future reaction of its competitors and the consequences upon it. In the first works on oligopoly, Cournot (1838) and Bertrand (1883), write about the elements used by the theory of games for analyse of the imperfect competition. The theory of games uses a general method of analysing the strategic interaction situations. This theory applies on analysing the companies' strategic behaviour of the Single Market, starting with the general framework of a game situation.

The game situations are based on few important elements: game rules; strategies to follow; earnings and in oligopoly situations we have: rules of the oligopoly game; strategies of the oligopoly game; earnings of the oligopoly game. The rules of an oligopoly game are made starting with the features of the economic, social and politic environment of the oligopoly market, with the laws of the trade practices. One of these rules regards the number of players meaning the number of companies operating on the market. The rules of an oligopoly game represent all the possible actions of each player. This one makes a complete list of game strategies. The possible strategies in an oligopoly game can be: prices increase, cutting prices or keeping them at the same level; increase, decrease or keeping the production at the same level; more, less or the same advertising; improving, lowering or maintaining the quality, etc. The earnings of an oligopoly game may be represented by the economic profits or losses of each company<sup>187</sup>. The earnings of the companies depend on their strategies and on their constraints which they deal with.

Studying the situations of imperfect competition, especially the *Single Market* where the buyers' decisions are interdependent, can be achieved with the help of non – cooperative games. Important applications of the game theory reside in different aspects of the oligopoly competition, for example: secret agreements or price forming study in a closed economic system. The situation of oligopoly competition, the companies can't deal with an unreceptive environment. The interdependent situations between different centres of decision and their contribution to obtaining a credible solution can be done with the help of *non-cooperative games* which developed a lot over the last years.

For the duopoly games, the possible strategies the two companies can follow are: agreement for profit division; or breaking the first agreement, thing that would allow the cheating company to obtain greater profits than its partner. This kind of structure is similar to the one of the game named prisoner's dilemma<sup>188</sup>. In case of a balance situation the two companies break or follow the duopoly agreement, they will sell at the same price, producing the same volume and obtaining the profit of a monopolist.

The theoreticians of the game with practice in economy showed that price war can be considered to be the result of a repeated duopoly game. The producers follow the concluded agreements until the request variation determine the price cut under a certain limit. The companies' reaction to the price cut is like it would be the result of a problem the competitors deal with. They have to operate in order to make the competitors believe in the penalty in case of agreement breakage. Updating the credibility of the menaces is important in order to respect the agreement as longer as possible. On this kind of markets, from time to time, price war can occur, which can end with the exclusion from the market of the weaker competitors.

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<sup>187</sup> Kreps D. M., Théorie des jeux et modélisation économique, Ed. Dunod, Paris, 2002, p.69.

<sup>188</sup> Jhele G. A., Reny P. J. – *Advanced Microeconomic Theory* - Ed. Addison Wesley, New-York, 2006,p.231

### 3. The Bertrand model

Bertrand (1883) suggested a solution which depended on price variation; he started with a rather simple case where two companies produce the same product, settling their selling price. The production costs for each product are constant (marked with letter „c”) and identical for both companies. In this case the consumers will buy from the company which has lower prices. We suppose that the two companies share the total request in two, if the prices are the same.

Bertrand’s paradoxical result is that at an equilibrium level the price is equal to the marginal cost and the profits are null. None of the companies can improve the profits because it would obtain a negative one cutting the price. If a company settles a price higher than the marginal cost, the other company will be interested in settling a lower price in order to cover the entire request. The question is if Nash equilibrium is present where the price is equal to the marginal cost. The Nash equilibrium, if present, is it unique?

In the Bertrand model, Nash equilibrium is a combination of strategies weakly controlled: some players are not interested in settling an equal or higher price than „c” (the cost), others set a price equal to „c”<sup>189</sup>. An irrational player who sets a price (P) higher than the cost (c) he obtains the same profit as the player who will obtain Nash equilibrium if the competitor always sets a price (P) equal to the cost (c). The player’s irrationally occurs when a competitor must explore such behaviour, choosing a price ( $P_1$ ) higher to the cost but lower to P. This kind of argument shows that players can be interested in manipulating their competitors making them believe that they are irrational. If player 1 is an agent with “irrational” reputation, choosing a monopoly price, then player 2 sets a price  $P_2$  lower to the monopoly one. But he is not sure that player 1, declared irrational but being rational benefits by his reputation, then he will chose a price lower to  $P_2$ .

Such reputation problems can be studied in a dynamic framework. We will consider the reputation game for two companies of the Single Market, marking with  $\delta$  the discount factor and trying to settle the perfect Bayesian equilibrium. The game can run by converse induction taking into account the beliefs. If the cooperation was supported in the first period, then in the final period the players will have the same reputation. If the players broke the agreement then they would lose the initial reputation. But choosing a cooperative strategy the player maintains the reputation, without bringing any new information regarding his behaviour, meaning the cooperative one. If we assume that the players cooperated in the first period then in the last one, a player obtained  $\alpha\delta\pi$  this one being a price lower than the monopoly one. The question which rises is: are the players interested in cooperating at the beginning? The company has an updated hope for earning equal to  $\alpha(\pi/2 + \delta\pi)$  in case of cooperation. In case of non cooperation for the entire monopoly profit the company will obtain  $\alpha\pi$ , losing its reputation. The company is interested in cooperation when  $\delta \geq 1/2$ . This model shows that the companies can be interested in cooperating at the beginning when their option for the present is low enough. In this case, we can talk about cooperation at the beginning of the game which degenerates in a price war. The reputation phenomena can be very important if there is the slightest incertitude regarding the perfect rationality of the players. Such a feature can explain many behaviours assuming that the players believe there are other “irrational” players but with a lower probability.

The examples mentioned above express the result of the strategic interaction which is very sensitive to the companies’, products’ and markets’ features. There are many different situations by which the consequences of these features can be studied (for example: the problems related to noticing the products’ quality, the advertising, the strategies of research and development, the network of distribution, the possibilities of market input and output). If we start with a normal game ( $x_1, \dots, x_n; u_1, \dots, u_n$ ) we will assume that the players behaviour is decentralized, each of them having to chose alone a strategy, ignoring any decision of other players. The communication

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<sup>189</sup> Kreps D. M., *Théorie des jeux et modélisation économique*, Ed. Dunod, Paris, 2002, p.136.

possibility between players is excluded. There is no initial result or a game history which could emphasize the strategies used more frequently than the one of the others. We can take into account all the strategies as being a priori equally possible and also the choice of the best strategy.

In order to determine a *non - cooperative* behaviour the controlling strategies can be removed. Strategy  $Y_i$  of player “i” is controlled by strategy  $x_i$  (where the results  $(x_1, \dots, x_{i-1}; y_i, x_{i+1}, \dots, x_n)$  represent a non – controlling assembly of strategies of player “i”) when whatever would be the strategic choice of other players, the use of  $x_i$  is at least as profitable for “i” than the use of “ $y_i$ ”, meaning strategy “ $x_i$ ” better than  $y_i$ . The games which have an equilibrium regarding the controlling strategies will be the decentralized solution of the non-cooperative game.

The strategy used by the company of the Single Market depends on the forecasts made on the reactions of the competitor companies, its analysis can be achieved with the help of the “game theory”. In this game each player must choose between a *pacifist strategy* and an *aggressive strategy*. The results of this game are described in picture nr.1.

		$Y^*$	
		Pacifist	Aggressive
$X^*$	Pacifist	(2, 2)	(0, 3)
	Aggressive	(3, 0)	(1, 1)

Fig. 1 — Game matrix of the Single Market.

In picture 1 each of the four boxes in the game matrix represent a result of the game with use of  $X^*$  in left and use of  $Y^*$  in right. We can verify if the pacifist strategy of a player is controlled by his aggressive strategy. In our situation the game has only an equilibrium in controlling strategies (aggressive, aggressive) which corresponds to an open price war of the Single Market. Such a result is not acceptable because the result “peace” meaning (pacifist, pacifist) is proffered by the two players in other words the war is not an optimal Pareto. The dilemma of the game shows that for a player which is not entirely sure on the pacifist intentions of its partner the use of the aggressive strategies is needed to follow the individual interests but the mutual interest recommends that everything has to be done in order to obtain a pacifist result.

The dynamic approach of the economic reality goes to the change of parameters of the variables which economic and social life. The key to survive in case of economic games is the capability of the companies to adjust their strategies to the environment which is in continuous change<sup>190</sup>. A correct forecast of the future events it is necessary. The cross - impact analysis, the request - hazard forecast as well as other numerous scenarios are just a few of the methodologies used in economic forecasts.

#### 4. Conclusions

In the economy structural changes and oscillations are the rule and not the exception and the stationary states become instable when certain parameters vary. The economic actors have different behaviours on the Single Market which have different consequences depending on the number, relative size and strategies approached by the other economic actors. Starting with the rationality hypothesis when the agent’s contentment is affected by the others decisions the game theory defines solutions for solving the situations of conflict.

<sup>190</sup> Sirghi Nicoleta, *Microeconomie Aprofundata. Teorie si aplicatii*, Editura Mirton, Timisoara, 2008, p.92.

The economic level of the competition can be considered as a mechanism of resources allocation which allows, in many cases, the promotion of the economical efficiency. The game theory contribution to the development of the competition politics can solve some problems related to the price system and to the intelligence exchange. For this reason the notion of competition was and is related first to the behaviour hypothesis of the economical agents and second to the relative hypothesis of the market operation. When the coordination of the individual actions is adjusted by competition, each economic agent must forecast the other agents' actions and then maximise depending on the results their own behaviour.

*Conducting a study in a dynamic environment regarding the Single Market represents a very important problem of the contemporary economy. We are interested in the theoretical results which are revealed in literature regarding the theory of games starting with the key concepts of this one: games, strategies, balance, game value, etc. The research is focused on the main oligopoly market structures from the microeconomics point of view. For this reason it is an opportunity to study the structure of oligopoly type of markets with the aid of non – cooperative games.*

*The use of the game theory as reference framework regarding the representation of the economic agents' on different market structure opens the way for an expansive field of investigation. The problem of the economic agents is no longer. The problem of the economic agents is no longer conducting studies for the operation of the perfect competition markets but to analyse the means in which they can coordinate the decisions, in dynamic configurations in a competitive environment affected by risk and uncertainty.*

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