Since the production has been found to be an economical means for satisfying human wants, this process requires a complex industrial organization together with a large investment in equipments, plants and productive systems. These productive systems are employed to alter the physical environment and create consumer goods. As a result, they are consumed or become obsolete, inadequate, or otherwise candidates for replacement. When replacement is being considered, two assets must be evaluated: the present asset, the defender and its potential replacement, the challenger. Since the success of an industrial organization depends upon profit, replacement should generally occur if an economic advantage will result. Whatever the reason leading to the consideration of replacement, the analysis and decisions must be based upon estimates of what will occur in the future. In this paper we present the Mapi algorithm as a procedure for evaluating investments or for analyzing replacement opportunities.

Keywords: replacement/renewal policy, Mapi formulas, adverse minimum, defender and challenger plant, operating inferiority, one-more year test
JEL Classification Code: M10, M11, M20, M21

1. The replacement-renewal problem for firms
Due to the typical activities of industrial firms and their increasing need for mechanized and automatized productive systems, plants, machinery and equipment play a central role in investment decisions.
In fact, in the present context of international development, industrial firms tend to increase in size in order to strengthen their supply pool and to reduce the threats from the competition. On the one hand, this growth in size requires increasingly larger investments in industrial plants; on the other, trends in demand, the pressure from the competition, which reacts with contrasting strategies, and, above all, the important trends in scientific and technical progress, which heighten the obsolescence processes, make such investments in industrial plants riskier.
Management must thus continually evaluate with even more care the convenience of their plant investments; once the investments are undertaken they must monitor the prospects for future use in order to determine the economically useful life cycle, determining the optimal period in which to discontinue operating the plant.
In determining the economic viability of medium-term industrial plans, calculations regarding the optimal duration of the plants are taking on more importance. Such calculations estimate the period of time beyond which the continued utilization of a plant, already installed and operational, is no longer economically viable and should thus be replaced or shut down.
Regarding the size of the invested capital, an appropriate replacement/renewal policy is one of the most important and thorough decisions for industrial firms, precisely in order to maintain the maximum economic efficiency in the productive structure, thereby improving quality and increasing productivity.
without increasing fixed unit costs.
If we consider the firm as a *viable system*, adopting Beer’s sense (Stafford Beer, 1972) of the term, or as a system of efficient transformation, in Mella’s sense (2007), it is clear that the decision to replace the plants is fundamental for maintaining the conditions of vitality and, more generally, for the necessary economic operating efficiency.

The replacement problem does not only concern plants and other fixed assets but is much more general: in fact, it can refer to the same businesses in which the plants participate.
In fact, as has been highlighted for years by the BCG (www.bcg.it), every business, after the start-up, boom and maturity phase, begins a period of decline.
The manager must then determine the optimal moment not only to discontinue the business in decline but also to replace it with another business, in a typical operation of the renewal of the business portfolio over time, thereby giving rise to a true *chain of renewal*.
Obviously the renewal of a business depends on the availability of other replacement businesses, and the moment for renewal is conditioned by the activation time of the new business.

*We can thus view a business* $B_n$ *as a link in a chain of business renewals in which its life cycle is not determinable in an absolute way, but only in relation to the business* $B_{n-1}$ *that precedes it and* $B_{n+1}$ *that replaces it (fig. 1).*

Prolonging the life of $B_n$ means delaying the start-up of $B_{n+1}$; calculating the optimal replacement time for $B_n$ would thus mean optimizing the profitability and cash flow of the composite business $B_n + B_{n+1}$.

However, the same reasoning must apply as well for $B_{n+2}$, and so on; this means there are no businesses that succeed each other over time with an autonomous existence, and autonomous profits and financial flows, but rather a chain of businesses where the life of each depends on that of the others.

![Fig. 1. The business chain](image)

The above discussion leads to an important implication: from the financial viewpoint the time factor can be recognized on the basis of two linked elements: timeliness in starting up a business and, at the same time, timeliness in ceasing operations. The former occurs when a firm perceives (or generates on its own) the potential for new demand; being first in the business means being pro-tempore in a position of near-monopoly and creating a competitive advantage until the time when competitors come on the scene.

This type of timeliness leads to notable advantages, among which: being the first to gain a consistent market share (Smith & Reimartson, 1991), setting high prices before the arrival of competitors, fidelizing the clientele that is served first, fully exploiting the learning of costs in order to gain internal competitive advantages, anticipating the recovery of fixed costs by reducing the risk on invested capital, and obtaining good profitability, which improves the firm's credit capacity for future growth.

### 2. Plant replacement

The renewal problem can specifically refer to the calculation of the optimal date for replacing the industrial plants, understood as the *set of instrumental goods in a productive system, which are*
necessary to carry out a given business.

The economic value of the industrial plants varies over the life cycle, and this variation depends on several factors, all of which are linked to the physical wear and tear and the economic obsolescence due to technical progress of both the plant as well as the product obtained with its help. These factors act to modify the firm’s profitability, which tends to gradually diminish, thereby necessitating the decision to replace the plants.

The calculation of the optimal operating period for a plant \( E \), not included in a chain of renewals, is thus a typical economic-financial problem: it is necessary to determine how much time is needed to cover the purchase and operating costs for the plant with the sales revenue of the production obtained from its use, with the desired operating result.

The problem could be solved through Break-Even Analysis; since the life of industrial plants is usually several years, the financial present value of the future costs and revenues relative to the purchase cost must be taken into account.

It is also necessary to consider that the length of the operating period of the plant itself conditions the costs and revenues that derive from it, in that:

- the purchase cost of a plant equivalent to the one to be replaced presumably varies from year to year since, due to technological progress, the retired plant is replaced with one technically more advanced and with greater productivity;
- the cost and revenue flows are sensitive to the period of reference, because of both the volume of demand and the prices, if the more modern plant allows the same products to be obtained at lower costs, or products of higher quality and with more attractiveness for the market;
- the disposal value of the retired plant varies according to the year it is shut down.

The replacement policy does not concern the single firms but rather an entire productive system, since the totality of replacement policies of the various firms is in turn a factor that determines the level of activity of the productive sectors involved in manufacturing the plants, with effects that can extend to the entire economic system and to the latter’s growth.

When firms begin to postpone the replacement of their plants, the immediate result is a decline in production and/or innovation regarding these industrial plants and this process – being part of a productive network (Mella, 2009) – can lead to a general crisis.

3. Conditions for replacement

Since for industrial firms the costs of plants represent a considerable portion of production costs, replacement policies play a key role in industrial plans.

In general terms, we can observe that renewal is not a simple operation that concerns only a single plant, but one that involves the entire productive system.

Especially with regard to complex plants, on which an entire business is founded, replacement involves all the plants that make up the productive network, since the introduction of a new and modern plant at a point in the network makes the other plants connected to it technically obsolete.

The renewal of large plants also involves implications, often relevant and radical, for the entire production, commercial and administrative organization, so that the replacement must be linked to measures that concern the firm as a whole.

Considering the time frame of the operation, we must always keep in mind that the convenience of undertaking renewal depends on the moment when the operation must be realized. Only if this occurs at the supposed optimal moment, from the economic point of view, will it allow the firm to maintain and improve its existing economic equilibrium or its re-establishment, when this is more or less seriously compromised by obsolete plants.

Negative consequences always result from carrying out renewal too late or too early with respect to the optimal moment.

When the replacement is delayed the firm will most likely see its production costs increase at an increasing rate and will not be able to remain price competitive with respect to its competitors, the more
so if the latter use more modern and efficient plants.  
If the delay of the renewal is too long, the business connected to the plant will enter its declining phase, becoming “dog”, so that it will have increasing difficulty raising the necessary capital for replacement, which is more urgently required due to economic necessity.  
Similar problems would be encountered by anticipating the replacement since, due to the contraction of the pay-back period, the cycle of economic utilization of the previous investment would not be completed, and the capital that is invested and not yet written off through the depreciation process cannot easily be fully recovered through the sale of the retired plant.  
Even from these simple considerations it is clear that management must place the renewal problem in the context of overall corporate strategy, understanding in a timely way the evolution of the organizational and environmental forces that influence its business and, more generally, its overall business portfolio.  
If it is necessary to be attentive at the moment of replacement, this means that the optimal utilization period for a plant also depends on the characteristics of the rival plants that can be considered as alternatives.  
From the above considerations it follows that, more than referring to an individual operation, renewal must be viewed more broadly to include the concept of “chain of renewals” in which the replacement of each link in the chain inevitably ends up conditioning all the subsequent links and, as a result, the entire chain of subsequent substitutions.  
We inevitably associate with the concept of a chain of renewals that of obsolescence, viewed as a continual process of technological progress of the new plants with respect to those in use, both as regards productive efficiency, through an increase in productivity, as well as manufacturing quality.  
Precisely because of the continuity of technological progress we need to take into account this important variable when making renewal decisions, since such choices involve the entire future chain of renewals and not merely a single replacement of a plant.  
The first attempt in this direction comes from Terborgh and the Mapi formula he developed.  

4. The MAPI formula  
In 1949 the Machinery Allied Production Institute in Washington set up a study committee chaired by George Terborgh to come up with a new evaluation system for investments aimed at the equipment-replacement problem, which led to a general rule, usually referred to as the MAPI formula, that allows for the determination at a given moment of the opportuness of replacing an existing plant with a new plant that has come on the market, called the potential challenger plant.  
Even if the MAPI formula has been the focus of criticisms, it offers a general solution to the equipment-replacement problem that covers a wide range of cases.  
“Essentially, it provides a quick test of current replaceability on the basis of a prefabricated structure of assumptions and projections into which the analyst can insert the necessary stipulations and estimates for the case in hand (the amount of the investment, the income tax rate, the interest rate, the service life, the terminal salvage ratio, etc.). It is an elaborate gadget to simplify an otherwise almost insoluble problem.” (Terborgh, 1956: p. 138).  
The novelty of the potential replacement plant should be understood both in terms of its complete technical efficiency – since it has not yet been utilized – as well as the improvement it embodies due to technical progress. With particular reference to the latter, Terborgh assumes continual technological evolution, which posits that progress is continuous and capable of making available to the firm a plant perfected at every future instant (in an abstract sense). There is thus the assumption of a continued operational superiority of future plants with regard to those the market offers and will offer; or, conversely, a continual operational inferiority of the plant to be replaced with regard to the new ones.  

- Plant \( E_t \) in use at instant \( t \) is called the defender. The new plant, \( E_{t'} \), available at \( t' \), is called the current challenger. Compared to the challenger, the defender has an operating inferiority that
manifests itself in the following values:
- gross revenues, $E_t$, less than those obtainable from $E_{t'}$;
- annual costs, $E_t$, greater than those required by $E_{t'}$;
- consequently an annual margin, $E_t$, less than that obtainable from $E_{t'}$; this margin has a monotonic non-decreasing trend for each future $t > t'$.

If the defender’s margins have a monotonic non-decreasing trend year-to-year, then its operating inferiority will also have a monotonic non-decreasing trend.

From the theoretical point of view, if the cost of capital were zero then the optimal renewal policy would obviously consist in replacing the plant every year (even at every instant, given the hypothesis of the continuity of technical progress) in order to have the best plant at every moment. In fact, Terborgh writes: “Now we consider a hypothetical question. What would be the proper equipment policy if capital goods were available free? The answer is obvious. Equipment would be replaced with great frequency, generally once a year or more often. With this high turnover, we should have continuously a state of “perfect” or “total” mechanization, yielding at all times the very highest operating performance the technology is capable of”. (G. Terborgh, 1962).

If we abandon this unrealistic hypothesis and assume that every investment entails a cost for the invested capital, then it is realistic to assume that the cost of the capital invested in the defender plant decreases as its period of utilization increases.

Regarding the challenger, as the utilization period increases there are two opposing trends at work: on the one hand, the cost of the invested capital declines, but on the other the operating inferiority increases.

The optimal solution is to find the point where the sum of the values of these trends is at a minimum – Terborgh defines this as the “adverse minimum” – that is, the minimum of all the alternative magnitudes, each of which is adverse: “Unfortunately machines are not to be had for nothing. They cost money, a fact which precludes the attainment of this state of technological blessedness. For when their cost is taken into account, mechanical perfection can be no longer the exclusive goal of equipment policy. The analyst has to choose between more capital cost with less imperfection or less capital cost with more imperfection. Now when we have alternative magnitudes, each of which is adverse, the best we can do is to find the proportion or combination of the two which minimizes the sum. This proportion is the key to correct equipment policy. It is the policy that minimizes the time adjusted sum or combined average of capital cost, and operating inferiority. This brings us to the concept of adverse minimum”. (G. Terborgh, 1962).

Two fundamental hypotheses are posited to formulate the evaluation criterion:
- All the future plants (the succession of challenger plants) have an adverse minimum equal to that of the best plant at the time of the survey. This is equivalent to hypothesizing not only the continuity of technical progress but also its constancy and uniformity over time. This serves to narrow the analysis to the data regarding the defender plant and to the best plant available for substitution at that moment, without worrying about what would instead happen if, due precisely to technical progress, the successive challenger should have very different economic-technical improvements.
- The operating inferiority of the defender increases linearly over time; that is, both the costs regarding the physical deterioration of the plant (costs for maintenance and lower productivity) and those linked to the plant’s obsolescence grow linearly.

These hypotheses reduce the analysis of the opportuneness of an investment in plant substitution to a comparison of the adverse minimum of the most technically-advanced plant (the challenger) and the plant in operation at that moment (the defender), based on the procedure indicated in the following section.
5. The calculation of the adverse minimum
We indicate by $E_0$ the defender plant and by $E_1$, $E_2$, $E_3$, etc., the challengers available for substitution in the subsequent years.
We indicate by $m_0$, $m_1$, $m_2$, $m_3$, etc., the annual margins (revenues less operating costs) that would occur under the assumption that after $t_0$ the best new plant is always utilized.
For convenience sake we assume that the margins are discrete and achievable at the end of each year.
If we set $t$ as the instant in which the plant $E_0$ begins operating, and setting $h = 1, 2, 3, \ldots$, then the initial annual margins for each new and improved plant will be:
$m_1(t+1)$, $m_2(t+2)$, $m_3(t+3)$, \ldots $m_h(t+h)$
and, based on our hypotheses, we will initially have:
$m_1(t+1) < m_2(t+2) < m_3(t+3) < \ldots < m_h(t+h)$
If the firm keeps plant $E_1$ in operation in the year subsequent to year one, when technically better plants are instead available which, through technical progress, can offer increasingly greater initial annual margins, it will achieve lower annual margins: in other words, the firm would sustain costs due to the obsolescence of the plant with respect to new ones.
If the firm should thus maintain $E_1$ in operation past the first year, it would achieve lower annual margins, even while plants with higher annual margins would be available.
If we indicate by:
- $m_1(t+1)$ the annual margin achievable with plant $E_1$ one year after it has entered into service at instant $t$;
- $m_1(t+2)$ the margin achievable with $E_1$ two years after it has become operational; ... 
- $m_1(t+h)$ the margin attainable with $E_1$ $h$ years after its adoption by the firm,
then we obtain the following relation:
$m_1(t+1) > m_1(t+2) > m_1(t+3) > \ldots > m_1(t+h-1) > m_1(t+h)$
The above relation indicates that if the firm should continue to utilize $E_1$ for periods subsequent to the first year, it would suffer disadvantages from lower margins owing to the gradual deterioration of the plant, together with:
- an increase in the operating costs due to a reduction in the technical productivity of the plant;
- a reduction in sales revenue from products which, due to the wear and tear on the plant, may have lower quality;
- an increase in maintenance costs.
We thus define the operating inferiority of $E_1$, for each subsequent year of utilization $(t+h)$, as the difference between the net margin of a new plant and that of the plant in consideration:
$m_1(t+h) - m_1(t+h) = g_h(t+h)$
The net present value of the operating inferiority, for a duration of “$n$” years of plant utilization, after adding the purchase cost $C_1$, and subtracting the revenue from salvage obtainable after “$n$” years, $S_1(n)$, at a chosen given discount rate, “$i$”, takes on the meaning of the total opportunity cost of the plant for “$n$” years:
$B_1(n) = C_1 - S_1(n) v^n + \sum_{h=1}^{n} g_h(t+h) v^h$ [1]
where \( v = (1 + i)^{-1} \) is the present value coefficient at discount rate “i”.

Using \([1]\), it is more significant to determine the \textit{average annual burden of service of the plant} in use for “n” years:

\[
b_1(n) = B_1(n) \frac{i}{1 - v^n} \quad [2]
\]

If we take “n” as a variable, we can calculate \([2]\) for periods of increasing length, \( n = 1, 2, 3, \ldots \), and determine the period “n*” that corresponds to the minimum average annual burden of service \([2]\). This value, \( b_1(n*) \), represents the \textit{adverse minimum} of \( E_1 \), “n*”.

However, since we are looking for the desirability of substitution of the \textit{defender} plant \( E_0 \), we must compare the adverse minimum of the challenger \( E_1 \) — that is, \( b_1(n*) = \min \) — with the \textit{annual replacement cost} of the defender \( E_0 \), called the \textit{adverse cost}, which is calculated as follows:

\[
\Delta_0(t) = \left[ S_0(t) - S_0(t+1) \right] + S_0(t) i + g_0(t+1) \quad [3]
\]

where:

\[
\left[ S_0(t) - S_0(t+1) \right] \text{ represents the loss in value of the scrap due to an additional year’s utilization of } E_0;
\]

\( S_0(t) i \) is a year’s lost interest on the \textit{terminal value};

\( g_0(t+1) \) is the operating inferiority of \( E_0 \) following its prolonged use for another year.

The defender \( E_0 \) should be replaced with the challenger \( E_1 \) when:

\[
\Delta_0(t) > b_1(n*) \quad [4]
\]

If this optimality condition is not satisfied then it is economically more useful to utilize \( E_0 \) for another year, at the end of which a new comparison will be needed, but this time between \( E_0 \) and \( E_2 \) for the following year.

If the inequality \( \Delta_0(t+1) > b_2(n*) \) does not still exist, then it will be necessary to utilize \( E_0 \) for another year and then compare \( E_0 \) and \( E_3 \).

The process should be repeated every year until the optimality conditions exist.

6. The MAPI formula as an approximative method

The procedure presented to this point is difficult to apply in practice due to the difficulties in estimating the \textit{adverse minimum} of the challenger plant relative to the length of utilization of the defender. In fact, a too forward-looking knowledge would be required, which is almost impossible to satisfy.

To overcome this difficulty Terborgh suggests a simplified formula, commonly known as the MAPI Formula, which allows us to determine the sum of the adverse minimums of the challenger plant (thus its adverse minimum), thereby reducing the calculation to a comparison of data regarding only the subsequent year.

In practice, starting from the assumption that the simple average of the \textit{operating inferiority} of the challenger for each period is higher than the corresponding equivalent annuity and that, on the other hand, the simple average of the cost of capital is lower than the corresponding average annual cost, financially calculated, then in order to avoid cumulative calculations for each year we can approximate the results by substituting the simple averages for the equivalent annualities that are theoretically required.

In fact, based on the assumptions made, the errors made as a result of this substitution are in opposite directions, and thus basically compensatory; since the sum of the simple averages is thus not that different than the sum of the equivalent annualities, then the adverse minimum of the challenger can
thus be approximated with a good degree of accuracy. Rather than calculate for each year of the probable life of the challenger plant the annuity equivalent to total opportunity cost, as is the case in [2], Terborgh, in order to calculate the adverse minimum, provides the following formula, which is easier and more practical to apply:

\[ b_1(n^*) = \frac{1}{2} \left[ (C_1 \cdot i - g_1) + \sqrt{2 \cdot C_1 \cdot g_1} \right] \]

[5]

Where, assuming a salvage value of zero and an interest rate “i”:
- \( C_1 \) is the purchasing cost of the challenger plant;
- \( g_1 \) is the average, constant over time (in case of the linearity of technical progress), of the operating-cost inferiority gradient; this is assumed to be equal to that of the plant in current operation;

The adverse minimum thus found is compared with the replacement cost of the defender plant, as shown in [3]. We thus return to the optimality condition [4], which determines whether or not renewal is appropriate.

7. An initial numerical example
A numerical example will clarify the application of the MAPI Formula given in [5]. Let us suppose the following data:
- the defender is operating for 12 years and has a residual value of € 20,000,000;
- after another year of operation the residual value is estimated to be € 12,000,000;
- the challenger, possessing greater technical efficiency, has an initial cost of € 200,000,000;
- the interest rate chosen is 10%;
- the challenger plant allows the firm to achieve, for the following year, the economic advantages indicated in Table 1, which compares the adverse minimums of the defender and challenger plants.

The value of the sum of the alternative minimums of the challenger plant is calculated by applying Formula [5]. Assuming a residual value of zero, a lower operating efficiency rate that is constant and equal to that of the defender plant, and inserting the following values:

\[ I_1 = 200,000,000; \]
\[ g_1 = 60,000,000/12 = 5,000,000; \]
\[ i = 0.10 \]

we obtain:
\[ b_1(n^*) = \frac{1}{2} \left[ (200,000,000 \times 0.10) - 5,000,000 \right] + (2 \times 200,000,000 \times 5,000,000)^{1/2} = \]
\[ = (44,721,400 + 7,500,000) = 52,221,400. \]

Since: 52,221,400 < 70,000,000, and thus the optimality condition [4] exists, it is absolutely appropriate to replace the plant in operation with the new one.
Table 1.

<table>
<thead>
<tr>
<th>EVALUATION ELEMENTS</th>
<th>ADVANTAGES FOR FOLLOWING YEAR IF THE CHALLENGER PLANT IS INSTALLED</th>
<th>ADVANTAGES FOR FOLLOWING YEAR IF THE DEFENDER PLANT IS MAINTAINED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower maintenance costs</td>
<td>3,900,000</td>
<td></td>
</tr>
<tr>
<td>Lower labor costs</td>
<td>34,500,000</td>
<td></td>
</tr>
<tr>
<td>Lower general production costs</td>
<td>25,000,000</td>
<td></td>
</tr>
<tr>
<td>Lower insurance costs</td>
<td></td>
<td>3,000,000</td>
</tr>
<tr>
<td></td>
<td><strong>63,000,000</strong></td>
<td><strong>3,000,000</strong></td>
</tr>
<tr>
<td>Inferior operating efficiency of the defender plant compared to the challenger plant for the following year</td>
<td></td>
<td>60,000,000</td>
</tr>
<tr>
<td>Loss in value of old plant</td>
<td></td>
<td>8,000,000</td>
</tr>
<tr>
<td>Passive interest on the residual value of defender plant</td>
<td></td>
<td>2,000,000</td>
</tr>
<tr>
<td>Sum of adverse minimums of defender plant</td>
<td></td>
<td>70,000,000</td>
</tr>
</tbody>
</table>

8. The new MAPI formula: one-more year test

With regard to the MAPI formula, Terborgh subsequently devised a new formula that, though based on the same principles as the preceding one, represents an extension and improvement over the latter. This new Formula does not stop at indicating the optimal time for renewal but is extended to consider all types of investment opportunity for the firm, and thus even new business projects, thereby making the replacement problem more closely adhere to the logic of the renewal of businesses, which was touched on in section 1.

The intent no longer is to examine the economic conditions of use of an individual machine or plant but to examine whether or not an entire productive system can be improved by replacing one or more parts of the “whole”, leaving the other parts unchanged.

The new MAPI Formula is set up to uncover the economic advantage that would accrue in the subsequent year’s income by investing a given capital in a “new project”, evaluating the qualitative-quantitative improvements to production.

The new formula is different, in both how it sets out the problem and the solutions it envisions. As far as setting out the problem is concerned, Terborgh partially modifies his initial assumptions. In particular, there is no longer the assumption of the linear growth in the operating inferiority of the plant in current operation. While maintaining the assumption of the constancy of obsolescence, it is recognized that, during the use of the plant, the trend in costs due to its operation and maintenance will not always be constant.

Terborgh maintains the assumption of the equality of the adverse minimums of the future challenger plants, on the one hand, and the best available plant on the market at the moment of the survey on the other, but this hypothesis is restricted in the sense that these plants must be similar: that is, have the same initial cost and the same residual value as the current plants when they are retired, after the same length of time.
A new procedure, called a “one-more year test”, is proposed according to which an *internal productivity rate* is determined for the subsequent year with regard to the net capital investment required to realize the project, and thus to replace the plants, which is compared to the hypothesis of not going forward with the renewal.

The algorithm provides an adjusted, after-tax rate of return criterion which the decision maker may use in replacement analyses or in selecting one investment opportunity from among many. The MAPI procedure is based on a one-year evaluation period. Alternatively, a period of several years might be considered with a “typical” single year obtained from an arithmetic average. Application of the MAPI formula requires the computation of six elements. These are used to calculate a rate of return for a one-year period based on average net investment. These elements are:

1. The arrival operating advantage, expressed as the sum of the net increase in revenue and the net decrease in operating cost. From this we derive that the *next year operating advantage* is composed of the increase in the operating result in the following year deriving from the project.

2. The initial net investment, expressed as the installed cost of the asset less any investment released or avoided.

At this point it is important to define several concepts.

– *Net invested capital.* This is composed of the capital needed to realize the project, and thus to purchase the new plant, install it, etc., net of the disposal value of the used plant and of any investment avoided due to the project’s implementation (costly repairs, improvements and change in the existing plants that would have been necessary without the new project).

– *Next year capital consumption avoided.* This is composed of the reduction in the residual value of the plant kept in service for another year if the new project is not implemented.

– *Next year capital consumption incurred.* This is the difference between the cost of the project and its value after one year, and in practice is equal to the amortization rate with interest of the new plant. Of all the above elements, *next year capital consumption incurred* is surely the most difficult to determine. The evaluations and forecasts required in this case extend, in fact, beyond the following year, since the annual share of costs for the investment project depends not only on the known amount of the purchase cost of the new plant but also on the residual value the plant will have at the end of the following year, and as a result on the estimated number of years of useful life. Thus, long-term forecasts are needed, which, nevertheless, can be avoided by using the diagrams developed by the MAPI formula, which allow the firm to determine a percentage which, applied to the net investment cost, allows it to calculate the value of the consumption of the required capital.

3. The terminal net investment obtained from the appropriate Mapi chart.

4. The average net investment, expressed as the sum of the initial net investment and the terminal net investment divided by 2.

5. The average capital consumption per year, expressed as the initial net investment minus the terminal net investment divided by the comparison period in years.

6. The increase in income taxes. The change in next year’s income taxes is basically the net increase in income taxes from the project in the subsequent operating year.

We can thus calculate the *internal productivity rate* for the following year – also called the “Mapi urgency rating” – with the help of a simple formula that permits a ranking of the investments, so that the most urgent ones (in the sense of having a higher relative productivity rate) can be realized first. The foregoing elements may be used to find the after-tax rate of return as:

\[
\frac{(1) - (5) - (6)}{100} = (4)
\]
This formula allows us to determine the return on investment from implementing the new plant; that is, the percentage of net income to the capital invested in the operation for the following year that the renewal will ensure.

At this point, in order to determine whether it is economically appropriate to renew the plant it is necessary to compare the rate of relative productivity, or the Mapi urgency rating, for the following year with the minimum productivity rate required by the firm to undertake the investment projects (Mella, 1997).

If the former is greater than the latter, then the investment to replace the plant should be undertaken.

9. A final numerical example
To better understand the new Mapi formula, it is useful to give an example from Terborgh’s Business Investment Policy. The problem involves determining the net profitability, expressed in percentage terms, of the investment in a new plant, equipped with devices that allow savings in manufacturing times, as a replacement for a similar existing plant.

The old machine is 39 years old and has a residual value of 200. It is assumed that paying a cost of 3,850 to adapt the old machine will provide another 10 years of use.

The overall cost of the new plant is 15,700 and it is assumed its useful life is 18 years and that it has a final residual value equal to 5% of the original cost. Linear amortization at constant rates is chosen, a tax rate of 50% of gross income is assumed, and normal or standard obsolescence (with a linear reduction of the annual margins obtainable with the plant’s use) is adopted.

The calculations and analysis from the above data are shown in Table 2.

### Table 2

**REQUIRED INVESTMENT**

1) Installed cost of project 15,700
2) Disposal value of assets to be retired by project 200
3) Capital additions required in absence of project 3,850
4) Investment released or avoided by project (2 + 3) 4,050
5) Net investment required (1 – 4) 11,650

**NEXT YEAR ADVANTAGE FROM PROJECT**

6) Assumed operating rate of project (hours per year) 3,300

<table>
<thead>
<tr>
<th>Effect of Project on Revenue</th>
<th>Increase</th>
<th>Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>7) From change in quality of products</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>8) From change in volume of output</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>9) Total</td>
<td>--- (A)</td>
<td>--- (B)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Effect of Project on Operating cost</th>
<th>Increase</th>
<th>Decrease</th>
</tr>
</thead>
<tbody>
<tr>
<td>10) Direct Labor</td>
<td>---</td>
<td>2,000</td>
</tr>
<tr>
<td>11) Indirect Labor</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>12) Fringe Benefit</td>
<td>---</td>
<td>350</td>
</tr>
<tr>
<td>13) Maintenance</td>
<td>---</td>
<td>1,160</td>
</tr>
<tr>
<td>14) Tooling</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>15) Materials and supplies</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>16) Scraps and rework</td>
<td>---</td>
<td>500</td>
</tr>
<tr>
<td>17) Dow time</td>
<td>---</td>
<td>425</td>
</tr>
<tr>
<td>18) Power</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>19) Floor space</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>20) Property taxes and insurance</td>
<td>---</td>
<td>150</td>
</tr>
<tr>
<td>21) Subcontracting</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>22) Inventory</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>23) Safety</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>24) Flexibility</td>
<td>---</td>
<td>400</td>
</tr>
</tbody>
</table>
**REQUIRED INVESTMENT**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>25) Others</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>26) Total</td>
<td>150 (A)</td>
<td>4.835 (B)</td>
<td></td>
</tr>
</tbody>
</table>

27) Net increase in revenue (9 A – 9 B) ---
28) Net decrease in operating cost (26 B – 26 A) 4.685
29) Next year operating advantage 4.685

**Non operating advantage**

30) Next year capital consumption avoided by project
   a) Decline of disposal value during the year ---
   b) Next year allocation of capital addition 422

**Total advantage**

31) Total next year advantage from project (29 + 30) 5.107
32) Total next year advantage after income tax (50%) 2.553,50
33) Mapi Chart – Allowance for project 785
34) Amount available for return on investment (32 – 33) 1.768,50

35) **MAPI URGENCY RATING** (1.768,5/11.650)x 100 15%

The value of 785 – that is, the share of capital cost – was calculated by recording from the appropriate monogram the percentage (4.7%) that must be applied to the original cost of 15,700 for a plant with a presumed useful life of 18 years, a disposal value of 5% of the original cost, and for which we assume that 25 out of each 100 of capital invested is obtained from credit operations with an interest rate of 3%, and 75 from stock issues that pay a 10% dividend.

This explains how the calculation was made to determine the affect of the plant renewal on the “following year’s” percentage of net income to capital invested in the operation.

**10. Critical considerations**

The critical considerations which can be made about the Mapi formula basically refer to the particular nature of the hypotheses underlying the problem that is posed; these do not always appear to conform to the actual operating conditions of firms. Among these we wish to point out:

- the hypothesis of the linear diminution of costs of future new plants; that is, the linearity of obsolescence and the evolution of technological progress at a constant rate over time;
- the hypothesis of the equality of purchase costs of future plants and those of the best plant currently available at the moment of the survey;
- the hypothesis that the comparison should be limited to the productivity of the plant currently in operation and the new plant only for the following year.

These hypotheses clash with the reality the firms operate in, since technical progress shows instead an uneven trend over time and the purchase prices of the plants in subsequent periods are not easy to predict.

Limiting ourselves to considering only the costs and revenues from the plant for only one year, ignoring the trend in costs and revenues from the plant in subsequent years, can lead to unreliable results if, in effect, such costs and revenues turn out to be different from those considered in the analysis.

Obviously using data from the industrial plans could help in extending the calculations for a longer period.

The model developed is thus able to produce valid results only in so far as these are limited to the field of application fixed in the initial hypotheses.
This must always be kept in mind in order to best adapt the model’s findings to the possible cases that arise in the actual management of the firm.

As Terborgh himself underscored, the formula does not solve all the renewal problems: “No knowledgeable person has ever claimed universality of application of the MAPI formula. It was conceived and presented as a replacement formula (the term “replacement” being broadly construed to include mixed replacement-improvement-expansion situations), its main purpose being to indicate the proper timing of re-equipment decisions. It was offered as an improvement over the primitive rules of thumb so widely employed in American industry to make these decisions. As such, it has proved very useful, as evidenced by the extent of its employment.” (Terborgh, 1956: p. 138).

**Bibliography**

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