ON RISKY BEHAVIOR IN BIMATRIX GAMES

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The experiment engaging 50 participants was performed to model and identify the determinants of the players' risky behavior. Here, a questionnaire and a bimatrix game containing the negative/zero payoffs were used to identify the players' motives to play risky strategies. Besides the concrete form of the payoffs, the individual risk attitudes were also proved to be statistically significant for risky behavior of the players.

Keywords: Bimatrix Game, Experiment, Risk, Dominant Equilibrium, Maximin, Laplace Insufficient Reason Criterion.

JEL: C9, C91, C92, D84

1. Introduction [1]

An increasing amount of attention has been drawn in recent years to the rapidly growing branch of economics science named Game Theory and Experimental Economics. It is partly because of the fact, that they incorporate concepts such as irrationality and uncertainty, dismissed or marginalized by the classical economics. It also offers explanations to various "real life" economic situations. To do so, it often uses a powerful tool – matrix games. Games, simplified human interactions and situations, expressed in this form allow for conduction of clear, simple but still valid experiments. Players are asked to choose from the set of strategies. We may study their choices in order to better understand the dynamics of decision-making. Decisions over strategies often converge to equilibrium, such as the one described by John Nash, where mutually best response strategies intersect (Nash, 1951). To determine other potential sets of strategies to which the players may converge in the game, we may use techniques offered by the decision theory (see Hansen (2005) for an overview) such as maximin (Pruzhansky, 2003), which alongside with the Nash equilibrium, we use extensively in this paper. Other very interesting approach in the field of decision under uncertainty is a Laplace insufficient reason criterion. The Laplace`s argument makes use of Jacob Bernoulli´s Principle of insufficient reason. The principle is that if no information is available about the probabilities of the various outcomes, it is reasonable to assume that they are equally likely (Pažek, 2008).

2. Model

Let $Q = \{1, 2\}$ be a set of players and sets $I = \{1, 2, \ldots, m\}$ and $J = \{1, 2, \ldots, n\}$ be their strategy space. The payoffs of player 1 and player 2 are defined by the matrices $\mathbf{A} = [a_{i,j}]_{i \in I; j \in J}$ and $\mathbf{B} = [b_{i,j}]_{i \in I; j \in J}$ respectively. Then, a two player game in normal form is defined as

$$\{Q = \{1, 2\}; I = \{1, 2, \ldots, m\}, J = \{1, 2, \ldots, n\}; \mathbf{A}, \mathbf{B}\}.$$

[1] Paper was supported by the grant VEGA No 1/0788/10

The strategies $i^* \in I$, $j^* \in J$ are the equilibrium strategies if for $\forall i \in I$ and $\forall j \in J$ the following holds: $a_{i,j^*} \leq a_{i^*,j^*}$; $b_{i^*,j} \leq b_{i^*,j^*}$. If the game has two or more equilibria denoted $(i^{'}, j^{'})$, and (i^*, j^*) is the equilibrium strategies pair for which $a_{i^*,j^*} \geq a_{i^{'},j^{'}}$; $b_{i^*,j^*} \geq b_{i^{'},j^{'}}$ is satisfied, then (i^*, j^*) are called the dominant equilibrium strategies.

Alternatively, players can choose not to play the dominant equilibrium strategies, but rather to maximize the minimum guaranteed payoff, using the maximin criterion. In this case, player 1 (row matrix player) will select the strategy

$$i^0 \in I; \max_{i \in I} \min_{j \in J}(a_{i,j})$$

and player 2 (column matrix player) will respectively select the strategy

$$j^0 \in J; \max_{j \in J} \min_{i \in I}(b_{i,j})$$

The third option for both players is to play according to the Laplace insufficient reason criterion, where both players calculate their expected payoff of their strategy taking into account the equal likelihood of each outcome within the chosen strategy. According to this criterion, player 1 will select the strategy

$$i^L \in I; \max_{i \in I} \left(\frac{1}{n} \sum_{j=1}^{n} a_{i,j} \right)$$

and player 2 will respectively select the strategy

$$j^L \in J; \max_{j \in J} \left(\frac{1}{m} \sum_{i=1}^{m} b_{i,j} \right)$$

Our experiment featured a two player game with the following payoff matrices for treatment HIGH RISK:

$$\mathbf{A} = \begin{bmatrix} 5 & -8 \\ 3 & 5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 6 & -9 \\ 4 & 5 \end{bmatrix}$$

For this pair of matrices according to our previous definitions the dominant Nash equilibrium strategies are $(i^*, j^*) = (1; 1)$. If players choose to maximize their guaranteed payoff, then they will select the strategies $(i^0, j^0) = (2; 1)$, and if they choose to play according to the Laplace insufficient reason criterion, then they will also select the strategies $(i^L, j^L) = (2; 1)$.
Treatment LOW RISK featured the following pair of matrices:

$$\mathbf{A} = \begin{bmatrix} 5 & 0 \\ 3 & 5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 6 & 0 \\ 4 & 5 \end{bmatrix}$$

Just as in treatment HIGH RISK, the dominant Nash equilibrium strategies are $(i^*, j^*) = (1; 1)$, the guaranteed payoff maximizing strategies are $(i^0, j^0) = (2; 1)$ and the Laplace insufficient reason criterion strategies are $(i^L, j^L) = (2; 1)$. Since guaranteed payoff maximizing and Laplace criterions lead to the same results, we will not distinguish them further on.
Note that in both pairs of matrices the combination of strategies $i = 1$ and $j = 2$ results in a loss compared to any other combination of strategies. Therefore we will call these strategies risky and the opposite strategies $i = 2$ and $j = 1$ safe strategies in both treatments.

3. Experimental setup

Experiment was conducted as a classroom experiment at the Faculty of Economics of Technical University of Košice in the fall semester 2010. The experiment was run using ComLabGames software, which is designed to conduct experiments with players over the Internet. Every session lasted approximately 90 minutes and was conducted during the seminar of the optional subject Game Theory. A total of 50 undergraduate students

participated on the experiment, of which 22 individuals participated on the treatment HIGH RISK and 28 individuals participated on the treatment LOW RISK. Among all players, 24 players were female and 26 players were male. Each treatment consisted of ten rounds. At the beginning of each session, supervisors invested 3 € into jackpot. Afterwards, each student was asked whether he wanted or not to participate on the jackpot. Supervisors then collected contributions of all the players that wanted to participate on the jackpot. The more someone invested into the jackpot, the bigger his claim on the final jackpot which included the 3 € invested by supervisors. The final payoff of each player was calculated by the supervisors according to his contribution on the jackpot and on his total score. Then, the players were asked to answer a short questionnaire, which was focused on the player's attitudes toward risk.
After, the experiment commenced. The software randomly and anonymously matched players into couples and then they started playing the matrix games. The payoff given by the matrices represented a score for the given round. After completing 10 rounds, the students were paid their final payoffs taking into account the total score they accumulated during the experiment.

3.1 Treatment HIGH RISK
In treatment HIGH RISK the risk cell (the combination of risky strategies) contains negative payoff points (-8,-9) for both players. The hypothesis is: the presence of the risk cell containing negative points reduces the frequency of risky strategies of players in spite of abandoning the dominant equilibrium and lowering the total possible score of both players.
In treatment HIGH RISK, 14 females and 8 males participated. Results of treatment HIGH RISK are visualized in Table 1. Results show, that in the majority of cases (48,18 %) the combination of safe strategies was selected, meaning that players rather avoid the possibility of losses by reducing their gains and playing according to the maximin or the Laplace insufficient reason criterion.

	$j = 1$	$j = 2$
$i = 1$	30,00 %	4,55 %
$i = 2$	48,18 %	17,27 %

Table 1: Results in treatment HIGH RISK

Among row players, the risky strategy occurred more often than among the column players. Figure 1 shows the frequency of the risky strategy among row players and column players.

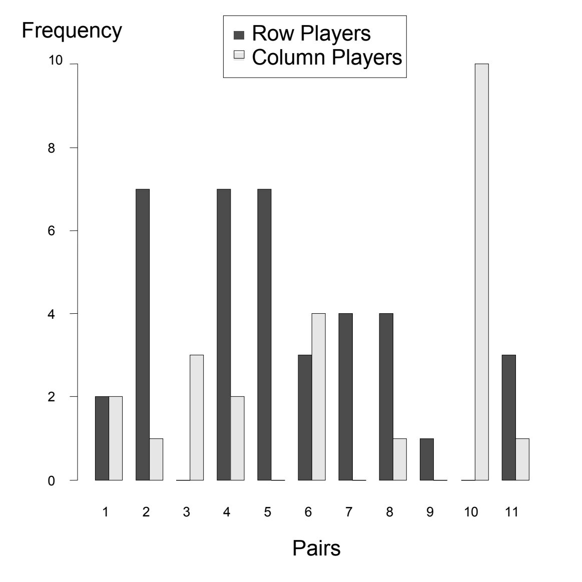

Figure 1: Frequency of risky strategy in treatment HIGH RISK

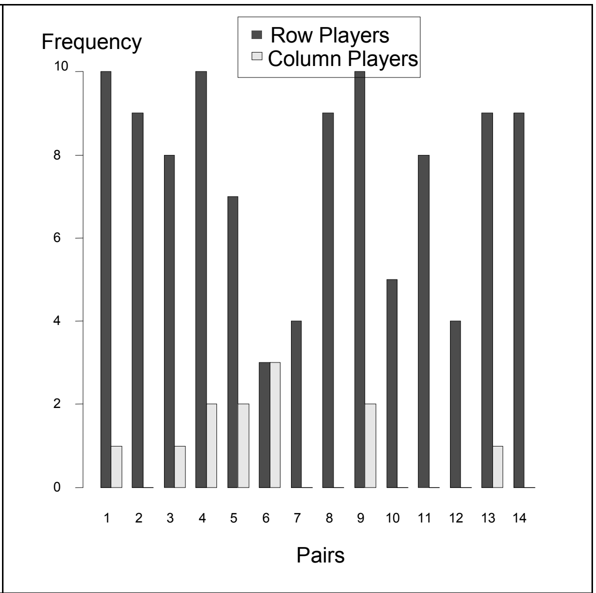

Figure 2: Frequency of risky strategy in treatment LOW RISK

3.2 Treatment LOW RISK

In treatment LOW RISK the risk cell contains (0,0) points for both players. The hypothesis is: since the risk cell contains 0 payoff points and not negative points for the players, it will not reduce the frequency of risky strategies compared to treatment HIGH RISK. Thus, players will be able to reach the dominant equilibrium more often than in treatment HIGH RISK.
In treatment LOW RISK, 10 females and 18 males participated. Results of treatment LOW RISK are visualized in Table 2. Results show, that in the majority of cases (67,86 %) the dominant equilibrium was reached. In other words, players played according to the dominant equilibrium criterion.

	$j = 1$	$j = 2$
$i = 1$	67,86 %	7,14 %
$i = 2$	23,57 %	1,43 %

Table 2: Results in treatment LOW RISK

Figure 2 shows how often the risky strategy occurred in the treatment LOW RISK. Compared to Figure 1, row players aren't risk averse any more.

3.3 Questionnaire

The questionnaire was aimed at determinating players' attitudes toward risk. First two questions concerned the sex and the number of siblings of the players. Third question investigated the players' rate of trust toward banks, savings-bank, insurance companies and stock markets. Fourth question asked the players, whether they would buy some good or service from brand they did not know. Fifth question asked the players, whether they would be willing to lend money to a related person without knowing the purpose of the loan. Sixth question inquired whether players had borrowed something in the past and haven't given it back by now. Seventh question asked the players if they have used the services of a betting agency at least once. Eighth question verified whether the players actively participate in charity. Ninth question asked the players whether they would be willing to abandon their financial reward in favor of someone else.

4. Results and Discussion

This study was focused on the investigation of risky investment strategies in the context of the investors' personal characteristics. Here, gender and number of siblings as well as three trust and risk factors were investigated.

Based on the empirical analysis, in case of treatment LOW RISK female players used the risky strategy more frequently than male players (F=1,027, p=0,048). In treatment HIGH RISK this hypothesis proved to be statistically insignificant.

Investigating the influence of the number of siblings on the risky behavior of the players, proved that for the treatment HIGH RISK, selecting of the risky strategy was preferred by players having 1 sibling (F=3,115, p=0,003). In case of LOW RISK treatment, this distinction was not statistically significant.

The link between the preference of the risky strategy choice and the investor trust/risk attitude was also examined. In both treatments, the statistically significant differences in accordance to the mistrust factor "Investment into the unknown trade-mark good" (F=0,876, p=0,045 – HIGH RISK) and (F=3,098, p=0,049 – LOW RISK) were identified. If considering the trust factor „Active charity participation", the risky behavior difference was also statistically significant (F=1,089, p=0,034 – HIGH RISK) and (F=1,112, p=0,001– LOW RISK). Last investment factor „Using the betting agency services" was also proved to be significant in both treatments (F=0,845, p=0,009 – HIGH RISK) and (F=0,817, p=0,04 – LOW RISK).

5. Conclusion

Based on the above presented research results, we can conclude that individuals expressing their positive risky attitudes in real situations, preferred risky strategies more frequently even in the experiment in both treatments.

Up to now we were only considering part of the social background and personal characteristics of players. However, we also proved that player preferences considering the risky strategy differed based on the treatment (z= 3,53, p =0,002). This means that players do not play by the same criterion in both treatments, but rather change the criterion they play by depending on the treatment they are faced with. In treatment HIGH RISK they played according to the maximin or the Laplace insufficient reason criterion and in treatment LOW RISK according to the dominant equilibrium criterion.

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