# CORRELATIONS AND THEIR EFFECTS ON THE CREDIT PORTFOLIO

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*Combining risks does not follow the usual aritmethical rules, as it is the case with incomes. Totalling up two risks, each equalling 2, does not always have 4 as a result. The sum is usually smaller, due to diversification. The correlation-based calculation shows us that the sum will range between 0 and 4. This is the essence of diversification and correlation. diverisfication reduces the portfolio's volatility.*

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The losses generated by the credit risk do not manifest simultaneously.[306]
The portfolio's risk results from the individual risks of each of its components. The income brought by the credit portfolio equals the algebrical sum of the incomes generated by each individual loan. These incomes are defined as the amounts obtained from interest rates and fees produced by individual loans in a specific period of time. The distribution of possible credit values in the future is, in fact, the distribution of the above-mentioned incomes. The total income generated by the credit portfolio (based on the probability theory of distribution) does not depend on the correlations between credits. On the contrary, the portfolio risk, calculated by means of dispersion or volatility, will be more or less influenced by these correlations.
The individual risks of each granted loan and the correlations existing between them will determine the portfolio's risk as the result of diversification. It thus represents the difference between the aritmethical sum of individual risks ans the risk of the entire portfolio. Since the portfolio risk is very much influenced by the correlations between the risks associated to individual transactions, these will also play an important part in shaping the portfolio risk evaluation models.
We shall analyze the change in the portfolio value in a considered period of time, between the present moment ($t_0$) and another future moment ($t_1$). We are speaking about the time horizon necessary to ensure the portfolio risk management, usually one year for the credit portfolio evaluation models. This change depends on common factors for all the debtors and can refer to economic or industrial conditions.

The portfolio value for any loan $i$ in the considered period of time equals to:

$$\Delta V_i = V_{i,1} - V_{i,0} = V_{i,0} \cdot r_i$$

$r_i$ is the change rate of the initial credit's value. A negative value shows in fact a loss occured in its development.

For the credit portfolio's risk, changing the value of each initial credit represents a loss or a profit. The portfolio's value ($V_p$) is the sum of the individual values for all the granted credits (

306 Bessis Joel, „Risk management in banking", John Wiley & Sons, 2004, pag. 339-341

$V_i$) and the change in the portfolio's value is the sum between the changes in all individual transactions:

$$V_p = \sum_{i=1}^{n} V_i$$

$$\Delta V_p = \sum_{i=1}^{n} \Delta V_i$$

The credit portfolio's profitability in the interval $t_0 - t_1$ is calculated as it follows:

$$R_p = \frac{\Delta V_p}{V_{p,0}} = \frac{V_{p,1} - V_{p,0}}{V_{p,0}} = \frac{\sum_{i=1}^{n}(V_{i,1} - V_{i,0})}{V_{p,0}} = \sum_{i=1}^{n} \frac{V_{i,0} \cdot r_i}{V_{p,0}} = \sum_{i=1}^{n} W_{i,0} \cdot r_i$$

In this formula, the portfolio's profitability is the weighted average of the profitability rates of each credit, while the weights ($W_{i,0}$) are the initial ratios between the individual loan value and their sum. By definition, the sum of these weights is 1.

Distributing Loss for a Credit Portfolio

In the following example, we set two objectives:

-Showing the way in which the correlations between two credits influence the distribution of the portfolio's losses (we shal take a two-credit portfolio). The ratings of the two credits can be correlated, which also influences the loss distribution. The risk measurement is realized by means of specialized indicators, such as expected loss (EL) or loss volatility (LV);

-Calculating the probable common loss (PCL), depending on the conditioned or unconditioned correlation between the two credits.

In order to thoroughly evaluate the effects of correlation, we shall start with the analysis of a portfolio consisting of two uncorrelated credits. We shall encounter 4 distinct situations: neither of the credits brings losses, only one of them (A or B) brings losses and both of them bring losses. This division enables us to easuly analyze any change in the analysis of loss distribution as a result of correlation.

Our example will take into account a portfolio consisting of two credits (A şi B), whose loss risks can be inter-correlated. In the next table, we present the two loans, either conditioned or unconditioned by their loss probabilities.

**Table 1**

**The two-credit portfolio**

| | **Probability of non-reimbursement (PD)** | **Exposure** |
|---|---|---|
| A | 8% | 500 |
| B | 10% | 800 |
| $\rho_{AB}$ | 0% sau 20% | |

The loss distribution will take 4 values (as previously mentioned). Depending on the correlation or non-correlation between the two loans, we can have two distinct situations.

We shall start with the situation where the two credits are not intercorrelated ($\rho_{AB} = 0$). Building the loss probability distribution is quite easy, due to the fact that the losses that may occur with the two credits are not conditioned. We are obviously dealing with common loss probability (respectively for the entire portfolio).

In the following table, we calculate the common loss probabilities for the portfolio:

**Table 2**

**Calculating the common loss probabilities for a portfolio consisting of two uncorrelated credits**

| Unconditioned probabilities | | | | Conditioned probabilities $B\|A$ | Common probabilities |
|---|---|---|---|---|---|
| A | | B | | | |
| Loss | 8% | Loss | 10% | 10% | 0,8% |
| | | Profit | 90% | 90% | 7,2% |
| | | | | 100% | P(A) = 8% |
| Profit | 92% | Loss | 10% | 10% | 9,2% |
| | | Profit | 90% | 90% | 82,8% |
| | | | | 100% | P(B) = 92% |

The calculation of probabilities in the previous table is quite simple, as the occurence probabilities for the two credits are not interconditioned.

The main formula to create the table is:

$$P(A,B) = P(A) \cdot P(B)$$

The probability that the two credits bring losses is: $8\% \cdot 10\% = 0{,}8\%$.

The probability that A brings losses and B scores a profit is: $8\% \cdot 90\% = 7{,}2\%$.

In the following tables, we review the results obtained in the previous table, in order to emphasize the correlations between the two credits forming a portfolio::

**Table 3**

**The matrix of common probabilities: unconditioned losses**

| | | B | | |
|---|---|---|---|---|
| | | **Loss** | **Profit** | |
| **A** | **Loss** | 0,8% | 7,2% | 8% |
| | **Profit** | 9,2% | 82,8% | 92% |
| | | 10% | 90% | 100% |

**Table 4**

**The matrix of common probabilities: unconditioned losses**

| | **Loss** | **Total probabilities** | **Cumulative probabilities** |
|---|---|---|---|
| A and B - loss | 1.300 | 0,8% | 100% |
| A - loss, B - profit | 500 | 7,2% | 99,2% |
| B - loss, A - profit | 800 | 9,2% | 92% |
| A and B - profit | 0 | 82,8% | 82,8% |

The calculation of the expected loss (EL) and loss volatility (LV) results directly from the accumulated occurence probabilities of previously analyzed events.

The expected loss (EL) is the weighted average of all possible losses. The loss volatility (LV) equals the square root of the loss variance. It is calculated by totalling up the squares of differences between possible losses and expected loss (EL), all weighted with the corresponding probabilities.

**Table 5**

| | Loss | Total probabilities | $\Pr ob \cdot Loss$ | $\Pr ob \cdot (Loss - PA)^2$ |
|---|---|---|---|---|
| A and B - loss | 1.300 | 0,8% | 10,4 | 11.139,2 |
| A - loss, B - profit | 500 | 7,2% | 36 | 10.396,8 |
| B - loss, A - profit | 800 | 9,2% | 73,6 | 42.540,8 |
| A and B - profit | 0 | 82,8% | 0 | 11.923,2 |
| | | | **PA = 120** | **VP =** $\sqrt{76.000}$ **= 275,68** |

Subsequently, we study the situation in which the two credits are correlated ( $\rho_{AB} = 0{,}2$ ). We will have to calculate the common loss probabilities for the two correlated credits. Conditioned probabilities are different from unconditioned probabilities. Correlation increaes the common loss probability and the loss volatility.

The process of calculating the probability distribution for the correlated credit portfolio also uses the conditioned probabilities of the credit A's rating compared to the credit B's rating and vice versa.

This is why we shall define P(A=loss) = a, which means the unconditioned probability that A suffers a loss and P(A=profit) = 1 – a. The same goes for B ( P(B=loss) = b, P(B=profit) = 1 – b).

The probability of two such events (profit or loss) unfolding at the same time is calculated by means of the formula:

$$P(A,B) = a \cdot b + \rho_{AB} \sqrt{a(1-a) + b(1-b)}$$

$$P(A,B) = 8\% \cdot 10\% + 20\% \sqrt{8\%(1-8\%) + 10\%(1-10\%)} = 1{,}37\%$$

**Table 6**

**Calculating the common loss probabilities for a portfolio consisting of two correlated credits**

| Unconditioned probabilities | | | | Conditioned probabilities $B \mid A$ | Common probabilities |
|---|---|---|---|---|---|
| A | | B | | | |
| Loss | 8% | Loss | 10% | 17,14% | 1,37% |
| | | Profit | 90% | 82,86% | 6,63% |
| | | | | 100% | P(A) = 8% |
| Profit | 92% | Loss | 10% | 9,38% | 8,63% |
| | | Profit | 90% | 90,62% | 83,37% |
| | | | | 100% | P(B) = 92% |

We used the formula $P(B \mid A) = \dfrac{P(A,B)}{P(A)}$ to calculate all the other probabilities. The conditioned probability column shows the probability of credit A's influencing credit B.

Thus:

$$P(B = loss \mid A = loss) = \frac{P(B = loss, A = loss)}{P(A = loss)} = \frac{1{,}37\%}{8\%} = 17{,}14\%$$

$$P(B = loss \mid A = profit) = \frac{P(B = loss, A = profit)}{P(A = profit)} = \frac{6{,}63\%}{8\%} = 82{,}86\%$$

$$P(B=profit|A=loss)=\frac{P(B=profit,A=loss)}{P(A=loss)}=\frac{8{,}63\%}{92\%}=9{,}38\%$$
$$P(B=profit|A=profit)=\frac{P(B=profit,A=profit)}{P(A=profit)}=\frac{83{,}37\%}{92\%}=90{,}62\%$$

The other common progress probabilities for the two credits were calculated as it follows:

$$P(B=loss,A=profit)=10\%-P(B=loss,A=loss)=10\%-1{,}37\%=8{,}63\%$$
$$P(B=profit,A=loss)=8\%-P(B=loss,A=loss)=8\%-1{,}37\%=6{,}37\%$$
$$P(B=profit,A=profit)=90\%-P(B=profit,A=loss)=90\%-6{,}63\%=83{,}37\%$$

In the following tables, we review the results obtained in the two previous situations, in order to emphasize the correlations between the two credits forming a portfolio:

**Table7**

**The matrix of common probabilities: conditioned losses**

| | | B | | |
|---|---|---|---|---|
| | | **Loss** | **Profit** | |
| **A** | **Loss** | 1,37% | 6,63% | 8% |
| | **Profit** | 8,63% | 83,37% | 92% |
| | | 10% | 90% | 100% |

It is interesting to notice and comment on the differences between this case and the case when the credits were not correlated.

Thus, in the first situation ($\rho_{AB}=0$) the common loss probability was as high as 0,8% , while in the second ($\rho_{AB}=0{,}2$) the probability rose to 1,37%. This is explained by the fact that a factor which negatively influences a credit will have the same effect on the other credit.

The standard loss distribution with corresponding probabilities, inlcuding the negative ones, is presented in the following table:

**Table 8**

**The matrix of common probabilities: conditioned losses**

| | **Loss** | **Total probabilities** | **Cumulative probabilities** |
|---|---|---|---|
| **A and B - loss** | 1.300 | 1,37% | 100% |
| **A - loss, B - profit** | 500 | 6,63% | 98,63 |
| **B - loss, A - profit** | 800 | 8,63% | 92 |
| **A and B - profit** | 0 | 83,37% | 83,37 |

Further calculations will demonstrate that the expected loss (EL) stays unchanged, as it is not influenced by the degree of correlation between the two credits. In exchange, the loss volatility will get higher as the credits get more correlated. Anayway, in the end, we shall notice the advantages of diversifying credit portfolio.

The expected loss (EL) for the entire portfolio results directly from the loss distribution and it is correlated for the two credits in case as it is shown in the following table:

**Table 9**

| | **Loss** | **Total probabilities** | **Probability of loss** | $\Pr ob\cdot(Loss-PA)^2$ |
|---|---|---|---|---|
| **A and B - loss** | 1.300 | 1,37% | 17,81 | 19.075,88 |
| **A - loss, B - profit** | 500 | 6,63% | 33,15 | 9.573,72 |

| | | | | |
|---|---|---|---|---|
| **B - loss, A - profit** | 800 | 8,63% | 69,04 | 39.905,12 |
| **A and B - profit** | 0 | 83,37% | 0 | 12.005,28 |
| | | | **PA = 120** | **VP =** $\sqrt{80.560}$ **= 283,83** |

Therefore, the expected loss is the same in both cases and it equals the sum of expected losses for the two credits.

$$PA(A) = 0\% \cdot 500 + 8\% \cdot 500 = 40$$
$$PA(B) = 0\% \cdot 800 + 10\% \cdot 800 = 80$$
$$PA(A+B) = 40 + 80 = 120$$

The loss volatility (LV) was calculated as it follows:

$$VP^2 = (1300-120)^2 \cdot 1{,}37\% + (500-120)^2 \cdot 6{,}63\% + (800-120)^2 \cdot 8{,}63 + (0-120)^2 \cdot 83{,}37\%$$
$$\Rightarrow VP^2 = 80.560 \Rightarrow VP = 283{,}83$$

This value is higher than the one obtained when the two credits are not correlated (275,68).
The loss volatility will be lower than the sum of the two credits'volatilities, as long as the correlation coefficient is lower than 1.

The loss volatility for each loan is calculated by means of the formula: $VP = E \cdot \sqrt{p(1-p)}$, where E is the exposure and p the loss probability for the respective credit.

$$VP(A) = 500 \cdot \sqrt{8\%(1-8\%)} = 135{,}64$$
$$VP(B) = 800 \cdot \sqrt{10\%(1-10\%)} = 240$$
$$VP(A+B) = 283{,}83 < VP(A) + VP(B) = 135{,}64 + 240 = 375{,}64$$

As a result, the expected loss is not sensitive to the effect of diversification and the loss volatility (which gives us the portfolio risk after all) gets lower as the portfolio diversifies.
In the following table, we review the results obtained in the two previous situations, in order to put the highlight once more on the effects of the correlations between the credits forming a portfolio:

**Table 10**

**Comparing the statistical results**

| | $\rho = 0$ | $\rho = 0{,}2$ |
|---|---|---|
| Expected loss | 120 | 120 |
| Loss volatility | 275,68 | 283,83 |

**Conclusions**

We appreciate that the portfolio risk results from the individual risks of each of its components. The income brought by the credit portfolio equals the algebrical sum of the incomes generated by each loan. These incomes are defined as the sums obtained from interest rates and fees generated by each individual loan in a given period of time. The distribution of possible credit values in the future is in fact the distribution of these incomes.

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