PORTFOLIO RISK AND RETURN

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In this paper we will briefly present the main concepts and an application regarding the Portfolio Risk and Return. In the first part of the paper we will present some general theoretical concepts referring the theme, and in the second part, we presented an approach of a Portfolio Risk and Return, consisting of two assets, together with a practical application.

***Keywords**: Portfolio, Optimal Portfolio, Portfolio Risk and Return, Efficient Frontier*

***JEL Classification**: G11, C61*

Main concepts

A portfolio can be defined as an collection of investments owned by an individual or by an organization, which through the process of diversification, the risk may be reduced.
The optimal portfolio represents the portfolio with the smallest level of risk registered for a certain return. The portfolio risk is compounded by two main risks: systematic risk (cannot be avoided through the process of diversification) and the specific risk (it can be reduced through the process of diversification).
The systematic risk depends on various external factors, macroeconomic influences: average market interest rates, inflation rate, foreign exchange, political and social factors, economic cycle (recession or boom). The variability of the macroeconomic factors have an important influence over all portfolio return. The specific risks (may be reduced by diversification) depends by the nature of the issuer. For individual stocks, depends by the profitability of the company, debts ratio, market share, management staff, economic cycle of the sector/industry that the company activates, different current news or rumors regarding the company, etc.
The relation between the risk and return for individual assets, the global return of the portfolio comparing to the average return of the market, has been studied by numerous financial theorists, developing quantitative financial models. William Sharpe has developed a model referring to the correlation between the individual return of every portfolio asset (Ri) and the global return of the market (Rp)

$R_i \sim R_p$ and

$R_i = \alpha_i + \beta_i R_p + \varepsilon_i$ (Regression line) (1)

where: $\alpha_i; \beta_i; \varepsilon_i$ = parameters

$\beta_i =$ = regression coefficient (volatility) (2)
ε_i == specific parameter for the asset :i: (measures the individual risk)

Observation

The regression line is graphically presented bellow:

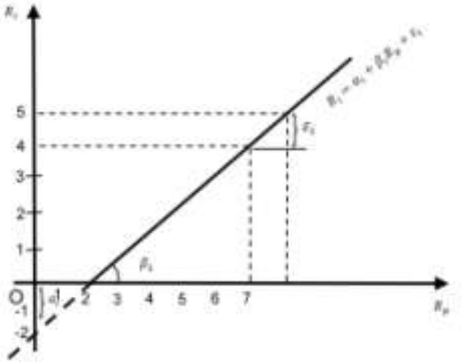


Fig.1 The regression line
(*Linear model of the Portfolio Return*)

2. Portfolio Risk and Return, consisting of two assets (particular case for n=2)

$\dot{I}$ = Investor
P = Portfolio consisting of two assets, t_1 and t_2, in amounts of x_1 and x_2 ; then
1) The Investor $\dot{I}$, constitutes its portfolio P, using the assets , t_1 and t_2
The capital invested (available funds) in t_1 andt_2 will be: (3)
$x_1+x_2=1$; with the following conditions $x_1 \geq 0,\ x_2 \geq 0$ and $0 \leq x_1 \leq 1 \quad 0 \leq x_2 \leq 1$
The investor's behavior regarding the assets in the near future, is given by the following elements:

$$t_1\colon\begin{cases}E_1;\\ \sigma_1\end{cases} \quad t_2\colon\begin{cases}E_2;\\ \sigma_2\end{cases} \quad şi \quad cov_{12} = \rho_{12}\sigma_1\sigma_2 \qquad (4)$$

where: $E_1=E_1(t_1)$ - average values expected
$E^2=E^2(t^2)$ - (or equivalent: mathematical expectation with the return on investment rate for the assets t_1 and t_2)

$\sigma_1 = \sqrt{\sigma_1^2}$
$\sigma_2 = \sqrt{\sigma_2^2}$ standard deviation for the assets t_1 and t_2
$cov_{12} = \rho_{12}\sigma_1\sigma_2$ – co variation between the return of the assets t_1 and t_2
ρ_{12} - Correlation coefficient between the return rates of the assets t_1 and t_2.
(5)

2) The mathematical expectancy of the return rate for the P Portfolio, can be written as:

$$E(R_p) = \sum_{k=1}^{2} x_k E(R_k) = x_1E_1 + x_2E_2 \qquad (6)$$

3) The dispersion in the rate of return for the P Portfolio is :

$$\sigma_p^2 = \sum_{k=1}^{2}\sum_{j=1}^{2} x_k x_j \sigma_{kj} = \sum_{k=1}^{2} x_k \left(\sum_{j=1}^{2} x_j \sigma_{kj}\right) = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2cov_{12} \ (7)$$

Observation:

O_1) Taking in consideration the values of the correlation coefficient ρ_{12} for the assets t_1 and t_2, we have the following situations:
1st Case: If $\rho_{12} = 1$, the assets t_1 and t_2 are perfect and positive correlated, and can be written using the relation : $\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_1\sigma_2 = (x_1\sigma_1 + x_2\sigma_2)^2$ (8)
So $\sigma_p = x_1\sigma_1 + x_2\sigma_2$ (standard deviation) (9)

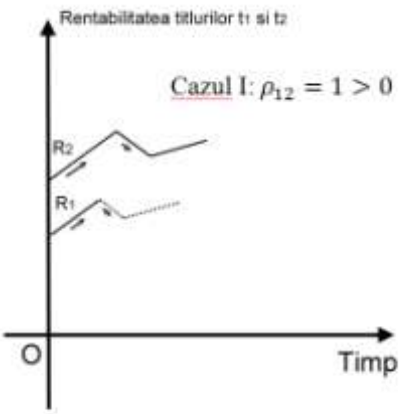
O_2) Taking in consideration that $x_1 + x_2 = 1$, and $E_p = x_1E_1 + x_2E_2$, we will obtain he following expression for the mathematical expectation of the Return Rate of Portfolio P:
$E_p = x_1E_1 + (1 - x_1)E_2$ (10)
From the last relation (10), we obtain:
$x_1 = \frac{E_p - E_2}{E_1 - E_2}; \quad cond: \quad E_1 - E_2 \neq 0$ (11)

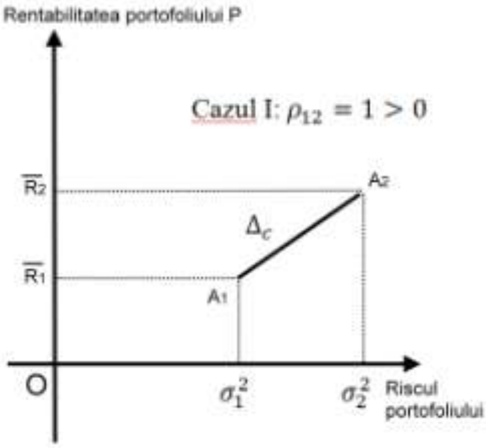
From: (10)→(11), we obtain the expression of the σ_p,:
$\sigma_p = E_p\left(\frac{\sigma_1 - \sigma_2}{E_1 - E_2}\right) + \frac{E_1\sigma_2 - E_2\sigma_1}{E_1 - E_2}$ (standard deviation for the case scenario of $\rho_{12} = 1$) (12)
O_3The same way we may treat the other cases :
2ND Case : $\rho_{12} = -1 < 0$, and :
3rd Case: $\rho_{12} = 0$.

4) Conclusions

Taking in consideration the correlation coefficient : ρ_{ij} (for the general case) or ρ_{12} (for the particular case of : n=2), between the return of the assets: t_1 and t_2, we may find 3 (three) types f extreme positive correlation ($\rho_{12} = 1 > 0$); zero ($\rho_{12} = 0$) and negative ($\rho_{12} = -1 < 0$).
Briefly, the three types of the correlation may be defined as:
1st Case : Strictly Positive Correlation ($\rho_{12} = 1 > 0$)

1. In case of increasing / decreasing the return of the asset t_1, there will be an increase / decreasing proportionally for the asset t_2





Graph no. 2 **Strictly positive correlation:** $\rho_{12} = 1 > 0$ Graph no. 3 **Correlation line** $\boldsymbol{\Delta_c}$

2. The Portfolio's Risk consisting of the two assets t_1 and t_2, in this case will be higher, so , on the correlation line, the Return-Risk line, (represented in the Graph no.3, on the line A_1A_2), will be the most performing assets

2nd Case: Zero Correlation: $\rho_{12} = 0 \Rightarrow \sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2$ (13)

In order to diminish the risk of the Portfolio P, consisting of two independent assets t_1 and t_2, there will pe an optimal (efficient) combination between the t_1 and t_2 components, that will get to the Portfolio of assets with minimum absolute variance, as may be seen in the Graph 4, below:

3rd Case: Strictly Negative Correlation: $\rho_{12} = -1 < 0$

1. In this case: $\rho_{12} = -1 < 0$, the correlation coefficient ρ_{12} it is equal with the inferior limit and
2. The rising (decreasing) of the first asset t_1 determines a decreasing (increasing) of the second asset t_2 (we are referring to the return of the investments)

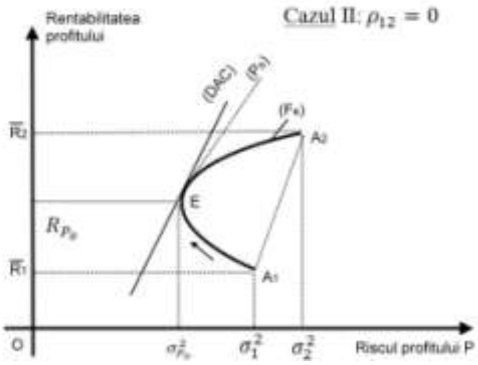


Fig. 4 **Zero Correlation** (for two independent assets)
Portfolio of assets with minimum absolute variance*: (PMAV*)

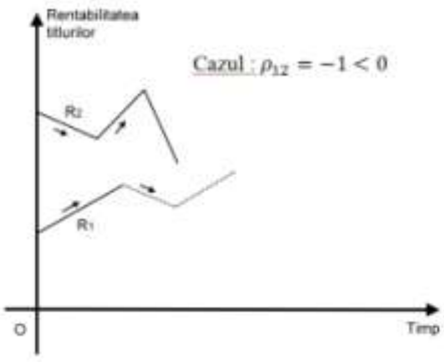
3. The Portfolio's (P) Risk, in this case scenario, is the smallest, because of an efficient combination of the two titles. This could allow even risk free conditions for the Portfolio mentioned above.

(to be seen : Point M that belongs to the "axis" of the Return (Profitability of P – Graph no. 6)

In this case, the Portfolio's Risk, becomes:

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 - 2x_1x_2\sigma_1\sigma_2 = (x_1\sigma_1 - x_2\sigma_2)^2 \quad (14)$$

And $\boxed{\sigma_p = \pm(x_1\sigma_1 - x_2\sigma_2)}$ (15)



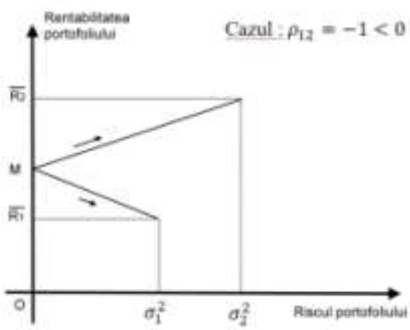


Fig. 5: Strictly Negative Correlation : $\rho_{12} = -1 < 0$
Fig. 6: Minimum risk, for the two assets mentioned

3. Application

For the given "P" Portfolio consisting of the certain assets "i" and "j", let's presume that: the returns of the asset given "I", corresponding to all the four trimesters of 2008 are:
$R_{i1} = 2$; $R_{i2} = 4$; $R_{i3} = 3$ and $R_{i4} = 1$
a) Calculate the parameters of the return distribution for the "I" title, and
$\overline{R} =?$ (Average return)
$\sigma_i^2 =?$ (Return dispersion)
$\sigma_i =?$ (Standard deviation)
b) The near future return expected? (its interval)

Answers:

We know from the hypothesis that:
$R_{i1} = 2$; $R_{i2} = 4$; $R_{i3} = 3$ and $R_{i4} = 1$ (return on the asset for the asset „i") than, according to the theory exposed above, we will have:

$$\overline{R} = \frac{R_{i1} + R_{i2} + R_{i3} + R_{i4}}{4} = \frac{2+4+3+1}{4} = \frac{10}{4} = 2{,}5$$

So: $\overline{R} = 2{,}5\%$ (average return (operated)) (16)

The dispersion return for the asset „i" may be solved with the formula:

$$\sigma_i^2 = \sum_{k=1}^{4} \frac{1}{T-1}\left(R_{ik} - \overline{R}\right)^2 \quad \Leftrightarrow \quad \sigma_i^2 = \frac{\left(R_{i1}-\overline{R}\right)^2+\left(R_{i2}-\overline{R}\right)^2+\left(R_{i3}-\overline{R}\right)^2+\left(R_{i4}-\overline{R}\right)^2}{4-1} =$$

$$= \frac{(2-2{,}5)^2 + (4-2{,}5)^2 + (3-2{,}5)^2 + (1-2{,}5)^2}{3} == \frac{5}{3} = 1{,}666 \ldots \cong 1{,}667$$

So: $\sigma_i^2 = 1{,}667$ (dispersion) (17)

Having the result for the dispersion return of the asset "I", the standard deviation will be:
$\sigma_i = \sqrt{1{,}667} \cong 1{,}291$ (standard deviation) (18)

Knowing the values of $\overline{R}$ and σ_i, results that:
The return of the asset „i" in the near future is expected to be between 2,5%$\pm$1,291% or equivalent to (1,209; 3,791) (19)

Conclusion: $\overline{R} = 2{,}5\%$ (average return)
$\sigma_i^2 = 1{,}667$ (dispersion of the return to asset "i")
$\sigma_i = 1{,}291$ (standard deviation) (20)
The return of the asset expected in the near future (2,5%$\pm$1,291%) q.e.d.

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