TIME EVOLUTION OF MATHEMATIC MODELS OF CURRENCY CRISIS

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The current period, characterized by a worldwide very significant economic instability, imposes an increasing attention focused on the currency crisis phenomenon. In this work we propose a review of the main approaches made on the mathematic models of the speculative attacks on the currencies.

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1.Introduction

Several economies of some states in Europe, South-East Asia or Mexico confronted during the last decade of the previous century with strong currency crisis, generated by the starting of several attacks with speculative character directed towards the exchanges controlled by the government. Depending on their chronological succession and on the way of tackling the problems, the models of the currency crisis divide in three categories: models of first generation currency crisis, models of second generation currency crisis, models of third generation currency crisis (or models of contagion).

2.Models of first generation currency crisis

The first generation models has been funded on the idea that the speculative attacks are generated by the inconsistency on long term of the governmental policies referring to the fix exchange rate. For example, the exclusive enhancement during a period of time of the currency amount, as an effect of the trial made by authorities for elaborating the monetization of a more accentuated fiscal deficit may cause the reduction of the international supply level, leading to the start of a speculative attack. In these situations, the government renounces to the fixed currency exchange and the currency depreciation takes place. These models studied the causes and the consequences of the currency crisis in Argentina (1978-1981) and in Mexico (1982). They proved that a deficit on long term of the external payments balance might lead to a decrease of the governmental supplies, making impossible the intervention on the market of the central bank.

The models of crisis of the payment balances were starting from the idea that the lack of balance in payments is the consequence of some too expansionist monetary and fiscal policies. The main indicators that defined the crisis of the payments balance were: the increasing high rate of the rising monetary amount, the decreasing rate of the state supplies and the budgetary deficit. In the conditions in which the private sector opts for the acquisition of foreign actives in exchange for the excess of local currency in circulation, the central bank will be obliged to recur to supplies, their level starting to decrease. If the central bank does not succeed in recurring to external loans in order to restore them, the supplies will continue to decrease, reaching a critical point in which the currency can fall down as a consequence of a sudden speculative attack.

Beyond the simplicity and the easiness with which they can be applied, the first generation models also present several weak points. Firstly, we cannot treat equally the behaviour of the public sector compared to the behaviour of the private sector. Thus, while the behaviour of the private sector (represented by speculators) is perfectly reasonable, this ones intuit very well the

subsequent evolution of the events, the public authorities have a rigid behaviour, being unable to produce quick reactions and measures on medium and long term which to protect the currency. Secondly, the concrete data of the currency crisis from the subsequent years infirmed the hypothesis that the main cause of the collapse of the exchange rates regime would be the insolvability. We observed that almost all the governments which were confronted to currency crisis at the end of the 80's and the beginning of '90's disposed of important currency supplies that allow them to defend their currency. In conclusion, the first generation models showed how in which the inconsistency of the macroeconomic policies of the government may occur a speculative attack on the fixed course of the currency.

3.Models of the second generation currency crisis

The analysis of the data offered by the currency crisis stressed upon certain lacks of the first generation models. Thus, with the help of these models, we could not explain certain crisis that appeared in the European monetary System, which were not confronted to significant fiscal or current account deficits. The second generation models analyse the crisis of the exchange rates regime produced without the existence of an excessive internal demand or an external lack of balance. These models are built on the waiting theory fundament. We suppose that the speculative attack itself can generate the devaluation of the currency, even in the conditions of a consistent policy of the fixed exchange regime. This thing significantly differentiates them from the first generation models. The production of the attack becomes thus a necessity, but also a sufficient condition for relaxing the governmental policy. The depreciation of the exchange rate appears as a natural consequence of this relaxation, making the attack become reasonable.

One of the most representative and mostly used model of second generation was realised by Obstfeld (1994). The model analyses the cost that the authorities have to bear if they renounce to the fixed rate, cost that will stimulate the government to maintain as much as possible this stability. The fixed rate will be maintained as long as the state will be able to cope with the inflationist pressures, being not forced to bear this cost. The first hypothesis is the opening of the national economy and the identification of the internal prices level at the same level with the price of the currency. The authorities aim to minimize the loss function:

$$L=(y-y^*)^2+\beta\varepsilon^2+C(\varepsilon), \quad (1)$$

where: y represents the level of the result (the production), y^* is the goal of the result for the government, ε represents the variation of the exchange rate (the price of the currency) and $C(\varepsilon)$ is the cost of maintenance of the fixed rate. For determining the "y" production we used the Phillips curve augmented on the basis of the expectances:

$$y=\bar{y}+\alpha\left(\varepsilon-\varepsilon^e\right)-u, \quad (2)$$

where: $\bar{y}$ represents the natural level of the result, ε^e is the variation of the exchange rate estimated by those who elaborate the internal prices policy, on the base of the advanced information and u is a shock, having the average zero. If $y^* > \bar{y}$, this one will determine a dynamic inconsistency which will have as effect the authorities' acceptance of realigning the cost of the rate. This dynamic inconsistency can be explained as follows: we suppose that ε^e is constant and the government chooses "ε" after they observe the shock "u". If a revalorization of the course takes place (that is a negative variation) the costs will decrease, tending towards a minimal value $C(\varepsilon)=c_{\min,}$ while the devaluation of the course (that is a positive variation) will determine the increasing of the costs towards the maximal value $C(\varepsilon)=c_{\max}$.

During the first step of the model, we leave aside the costs in the relationship (1), starting from the predetermined constant value ε^e. Replacing the expression of the production from (2) in the function of the loss (1), we obtain the value of "ε" for which the loss is minimal (the derived of the function "L" in rapport with "ε" is null). This value of "ε" replaced with the expression of "y" in the relationship (2) will lead us towards the production level expression y. Further, replacing

the expressions of "ε" and that of "y" in the loss function from the relationship (1), we obtain the expression of the loss in regime of variable rate:

$$L^{VAR} = \frac{\beta\left(y^* - \bar{y} + u + \alpha\varepsilon^e\right)^2}{\alpha^2 + \beta}.$$

If the state adopts the policy of a fixed exchange rate, then the variation "ε" of the rate will be zero and the loss in the regime of fixed rate is:

$$L^{FIX} = \left(y^* - \bar{y} + u + \alpha\varepsilon^e\right)^2.$$

In the second step of the model we also introduce the fixed cost $C(\varepsilon)$. In this case, the government accepts the devaluation of the rate only in the case in which the economic loss afferent to the fixed rate is higher than the one which corresponds to the flexible rate. So, the system will be viable only when the level of the shock "u" is high enough that $L^{VAR} + c_{max} < L^{FIX}$ or low enough that $L^{VAR} + c_{min} < L^{FIX}$. The revaluation, respectively the devaluation of the rate will appear when the shock "u" will be from outside the interval $[u_{min},u_{max}]$. The two extremities of this interval represent the limit values of the shock between which the state may maintain the fixed exchange rate. In order to effectively calculate the two values we solve the equation:

$$L^{VAR} + c = L^{FIX}.$$

In the two variants $c = c_{max}$ and $c = c_{min}$, after solving the equation from above we obtain:

$$u_{\max} = \frac{\sqrt{c_{\max}\left(\alpha^2 + \beta\right)}}{\alpha} - y^* + \bar{y} - \alpha\varepsilon^e \text{ şi } u_{\min} = -\frac{\sqrt{c_{\min}\left(\alpha^2 + \beta\right)}}{\alpha} - y^* + \bar{y} - \alpha\varepsilon^e. \quad (3)$$

We can further suppose that the shock "u" is uniformly distributed on an interval [-μ,μ], with μ>0. The government will abandon the fixed rate only in the case in which "u" is not situated in the interval $[u_{min},u_{max}]$. Considering the uniform distribution of "u" on the interval [-μ,μ] and the natural supposition $(u_{\min},u_{\max}) \subset [-\mu,\mu]$, we obtain:

$$M(\varepsilon) = \frac{\alpha}{\alpha^2 + \beta}\left[\left(1 - \frac{u_{\max} - u_{\min}}{2\mu}\right)\left(y^* - \bar{y} + \alpha\varepsilon^e\right) - \frac{u_{\max}{}^2 - u_{\min}{}^2}{4\mu}\right]. \quad (4)$$

We can determine the manner in which the reasonable expectation about the variation of the exchange rate $M(\varepsilon)$ is influenced by the variation of the exchange rate estimated by those who elaborate the internal prices policy ε^e. We derivate (4) in rapport with ε^e and we obtain:

$$\frac{\partial M(\varepsilon)}{\partial \varepsilon^e} = \begin{cases} \dfrac{\alpha^2}{\alpha^2 + \beta}, & u_{\min} > -\mu \\ \dfrac{\alpha^2}{\alpha^2 + \beta}\left(\dfrac{1}{2} + \dfrac{y^* - \bar{y} + \alpha\varepsilon^e}{2\mu}\right), & u_{\min} = -\mu \\ \dfrac{\alpha^2}{\alpha^2 + \beta}, & u_{\max} = -\mu \end{cases}$$

Because the first derivate of a function in a certain point represents the slope of the tangent at the graphic of the function from that point, the values obtained above describe the slope of the graphic of the function $M(\varepsilon)$ given by the relationship (4) on the three intervals taken into account. From the previous result we can observe that in two different situations the function $M(\varepsilon)$ from the relationship (4) has the same slope: for those values of the shock "u" higher than $-\mu$ and as well as when $u_{\max} = -\mu$. From the economic point of view we can interpret as follows: if ε^e takes high enough values so that the maximal accepted value of the shock arrives at the inferior limit of the interval [-μ,μ], and the expectations of depreciation of the rate will be similar to those from the case of the free flexible exchange regime.

In order to obtain the equilibrium condition, we calculate the fixed point of the function $M(\varepsilon)$ from the relationship (4). The state of perfect equilibrium supposes that the reasonable expectation regarding the variation of the rate in the following period $M(\varepsilon)$ coincides with the prevision of those who determine the price. Thus, the equilibrium will be obtained by solving the

equation $M(\varepsilon)=\varepsilon^e$. Replacing the expression of $M(\varepsilon)$ from the relationship (4) in the previous equation results the following equation:

$$\frac{\alpha}{\alpha^2+\beta}\left[\left(1-\frac{u_{\max}-u_{\min}}{2\mu}\right)\left(y^*-\bar{y}+\alpha\varepsilon^e\right)-\frac{u_{\max}{}^2-u_{\min}{}^2}{4\mu}\right]=\varepsilon^e.$$

In order to solve this equation we will introduce the expressions of $u_{\max}$ and $u_{\min}$ from (3), because $u_{\max}$ and $u_{\min}$ are them too functions of ε^e. We notice that the previous equation has several solutions, so there will be several states of complete equilibrium. The situation of the multiple equilibria is influenced by the structure of the national economic activities as well as by the preferences of the state. For example, if the private sector concentrates its economic preferences on small values, the state cannot engage the envisaged balance. The power of the fundamentals reflected in the state policy as well as the market structure have an important influence on the multiplicity of equilibria. An important conclusion of the above presented model shows that an event that had previously a small enough probability may have devastating consequences in the conditions of an incomplete structure of the economy, leading thus to the start of a self-fulfilling currency crisis.

4.Models of the third generation currency crisis – contagion models

In time, there were certain crisis propagated from a country to another, but this phenomenon was not due to substantial modifications of the fundamental macroeconomic data of the respective economies. It is in this way that we arrived to the phenomenon of contagion, explained by the mechanical behaviour. The third generation models are based on this concept appeared more recently in the specialty literature – the contagion. By contagion we understand the phenomenon of propagating or of correlating certain shocks between different countries, beyond the fundamental relationships between them and beyond the common shocks. From the analysis of these crisis we deduced the existence of some relationships between the price of actives and the exchange rate from a certain country and that of other emerging states. The appearance of these relationships can be explained, on one hand, by the action of the same factors or it can be the effect of some similar reasons as the policies adopted by the very developed countries, which can similarly influence the emergent states. These crisis are known in the specialty literature as "monsoonal crisis". On the other hand, a speculative attack from an emerging state can lead to modifications of the fundamentals from another emerging state: for example, a devaluation of a currency can imply the decreasing of the prices competitiveness in another country. This second type of crisis is called "spillovers". Thirdly, the crisis from certain countries can propagate abroad through channels too, as the way of perceiving the existing information or expectations and the opinion regarding the market, without influencing the fundamentals of the economy of the respective countries. The economic literature call these crisis simply – "contagion".

The models of contagion has at their base a model of currency crisis constructed by Masson (1998), starting with the crisis from Mexico (1994-1995) and Thailand (1997). These currency crisis generated powerful pressures on the exchange rates and on the prices of the actives on other emergent states' markets. Like his predecessors, Masson starts from the idea that a devaluation may take place when the currency supplies reach a certain critical point. The evolution of the fundamental macroeconomic variables may not necessarily have a certain tendency, but if the external debt surpasses a certain value then the shocks produced in the current account can determine the apparition of a crisis. The expectations for a crisis are evidenced in the loan costs paid to the external. Thus, the value of the external debt (presupposed, for simplicity, as being exogenous) becomes an essential variable for the existence of the multiple equilibria, as the high interest rates, due to the increasing of the debt costs, can push the supplies beyond the level that produces the devaluation.

The model includes two emerging states. The external environment (represented especially by the interest rate in the industrialized countries r^*) is supposed to be known. Firstly, the model has in

view the first country. We suppose that an external debt "*D*" expressed in domestic currency has been accumulated, for which we pay a variable interest, but for simplification we consider that there are not other net capital flows. Until a point that generates a crisis, the authorities finance any deficit (or surplus) of current account, determining some changes in the level of the supplies. The source of incertitude is represented by the shocks on the commercial balance "*T"*. If they are powerful enough as to determine at a certain moment "*t*" a reduction of the supplies R_1 under the critical level $\overline{R}$, then appears the devaluation. If S_1 represents the spot exchange rate at the moment "*t*" (the price of the currency) and S_{t+1}^{d} its value in the next period in the case of devaluation (thus we have $S_t = S_{t+1}$), then, the for the obligations in domestic currency, the ex-ante return on active is given by the relationship:

$$M_t\left[\ln\left(\frac{(1+r_t)S_t}{S_{t+1}}\right)\right] \approx r_t - \pi_t \ln\left(\frac{S_{t+1}^{d}}{S_t}\right) - (1-\pi_t)\ln(1+\delta) = r_t - \pi_t \ln(1+\delta) \approx r_t - \pi_t\delta,$$

where π_t represents the probability of producing a devaluation, „δ" is the proportional dimension of the expected devaluation and r^* plays the role of the free-risk exchange rate.

Moreover, we will further observe that the probability to produce a crisis will be also influenced by those expectations that lead to multiple equilibria.

The modifications appeared in the supply level will thus be given by the relationship:

$$R_{t+1} - R_t = T_{t+1} - (r^* + \pi_t\delta)D. \tag{5}$$

At the moment *t*+1 a crisis will emerge if:

$$R_{t+1} - \overline{R} < 0. \tag{6}$$

From the relationships (5) and (6) it results that the probability, calculated at the moment "*t*" for a currency crisis to emerge at the moment *t+1* will be:

$$\pi_t = \mathrm{P}_t[T_{t+1} - (r^* + \pi_t\delta)D + R_t - \overline{R} < 0 \tag{7}$$

Using the notations: $b_t = T_t - r^*D + R_{t-1} - \overline{R}$, $\alpha = \delta D$ şi $\Phi_t = \mathrm{M}_t(b_{t+1})$, we obtain

$$\pi_t = \mathrm{P}_t(T_{t+1} - r^*D + R_t - \overline{R} < \pi_t\delta D) = \mathrm{P}_t(b_{t+1} < \alpha\pi_t). \tag{8}$$

Expressing the probability π_t in terms of cumulative distribution in function of the variation of b_t, we may write:

$$\pi_t = F_\sigma(\alpha\pi_t - \Phi_t), \tag{9}$$

where F_σ is the cumulative distribution function of a normal distribution of variance σ^2. The relationship (9) defines the investors' formulation of expectations. Both members of the equation depend positively on π_t, the existence of multiple equilibriums will be possible. A necessary condition for this will be that $z = \frac{\alpha}{\sqrt{2\pi}\sigma} > 1$, which means that the slope of the cumulative distributive function is more inclined in a certain point than the left member of the same relationship. This can be also interpreted as a condition imposed on the level of the external debt and on the increasing of debts' level in case of a crisis, because $\alpha = \delta D$, referring to the standard deviation of the shocks in the commercial balance, "*σ"*.

We can also identify a condition on Φ_t, more precisely an interval of values for this one. The interval for the multiple equilibria is defined by two tangency conditions between the LHS and the cumulative distribution function, conditions obtained when the fundamentals are very good, respectively very poor. In particular, if we consider $w = \sqrt{2\log z}$, then the two conditions of tangency define for Φ_t the following interval in which are possible the multiple equilibria:

$$\alpha F_1(-w) + \sigma w < \Phi_t < \alpha F_1(w) - \sigma w. \tag{10}$$

The inequality (10) is in itself a condition on the supplies. If these ones surpass a certain value, then the crisis is lightly probable, while if they decrease under a certain minimum, the outbreak of a crisis is almost certain. Between these values Φ^{min} şi Φ^{max} multiple equilibria may take place.
In the second part of the model we present the relationships with other emergent markets. Further, the effects of spillover and the monsoonal effects, introducing the interactions between the reference country and other emerging economy by means of the competitiveness effects on the commerce. For simplicity, we suppose that all the structural parameters are identical in both countries. We will determine the probability of producing a devaluation in the first country depending on the probability of devaluation in the other country.
The model finally presents the three channels through which the crisis from different states may coincide in time. The monsoonal effects may take the form of the modifications of r^* (for example the rate of the interest in The United States) or of $\overline{S_t}$ (for example the exchange rate dollar-yen). The spillover effects can be represented by the modifications appeared in the initial level of the exchange rate in the second country. In the end, the contagion is illustrated by the expectations regarding the devaluation from the second country, which will directly influence the probability for a devaluation to emerge in the first country. The mathematic model described above has been successfully used for explaining the spreading of the speculative crisis in various emerging economies in South-East Asia in the period 1997-1998.

5.Conclusion

In time, a multitude of models of speculative attacks on the exchange rate were developed. The last works have dealt more and more with the research of the propagation of currency crisis starting from the similar characteristics and the interdependences between the countries.
But we cannot say that, together with the apparition of the contagion models, considered as being the most important third generation models, the problem of the speculative attacks on the currency of a country has been completely explained and solved. There are also a lot of aspects to be explained for elucidating the theory of situations of currency crisis, of the causes and the mechanisms of transmitting the crisis. The current financial and monetary environment, characterized by a great dynamism, is in a continuous transformation and we can expect it to offer us further examples that determine new research of the specialty literature representatives.

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