STOCK RETURNS AND THEIR PROBABILISTIC DISTRIBUTION (THE BUCHAREST STOCK EXCHANGE CASE)

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Abstract: Based on a long series of papers analyzing stock returns behavior we can speak generally about the stock exchange as a speculative market in the sense of the stable paretian hypothesis. Still, there are significant differences from a market to another and in many cases biases from normality are too insignificant in order to justify a radical change of approach. This radical change is less needed especially when the aggregating interval of price changes gets big enough, for example if we speak about weakly or monthly returns, cases in which the non normality hypothesis can be accepted in a comfortable way.

Keywords: stock returns, probabilistic distributions, stable paretian hypothesis

The Gaussian hypothesis regarding stock return distributions

There is a long tradition among the specialists from the financial area in considering that in speculative markets financial assets prices (especially stocks) behave in a manner similar to a random walk process. The random walk theory is based on the following hypothesis: changes in stock prices (and indirectly the stock returns) are independent random variables respectively changes in stock prices follow a certain probabilistic distribution. The first one to introduce a certain model for stock prices behavior (but also for commodities traded on designated exchanges) was Louis Bachelier in 1900[261]. The simplest and in the same time the most important model presented by Bachelier is built as following:

If $P_{(t)}$ represents the price of a stock at the end of the t period, then it is assumed that subsequent changes of this form[262] $P_{(t+T)}$-$P_{(t)}$ are independent random walk variables, which follow a normal law of distribution (Gaussian) with a null mean and a variance proportional with the T interval dimension.

Despite the fundamental importance of the process presented by Bachelier, model that is now known as "Brownian movement", the financial realities emphasize the fact that this model cannot explain in a satisfactory way the historical trading data accumulated in certain exchanges since the second half of the XIX century and this is due mainly because the empirical distributions associated with price differences such $P_{(t+T)}$-$P_{(t)}$ are too peaked in order to be compatible with Gaussian samples.

Still, in order to advocate the Gaussian hypothesis regarding price changes in financial speculative assets, [Osbourn, 1959][263] used a series of arguments based on the effects of central limit theorem from the probability theory. He rationalized as it follows: if price changes from a transaction to the other are independent and identically distributed random variables (iid) and they have a finite variance and the transactions are uniformly distributed in time, then the central limit theorem assures that price changes along a certain interval (day, week, month, year and son on) are normally distributed, being just sums of changes from a transaction to another. In other words, if the number of transactions gets bigger and bigger, aggregating random variables associated with the corresponding price changes will lead to random variables which tend to follow a normal law of probability.

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261 Bachelier, L., 1900, *Theorie de la speculation*, Gauthiers-Villars, Paris.

262 This model of Bachelier assumes in an implicit manner that the variance of the differences $P_{(t+T)}$-$P_{(t)}$ is independent from the $P_{(t)}$ value. There are some reasons to expect that the standard deviations associated with ΔP to be proportional with the price level and for this reason mainly, many modern authors suggested that the original condition regarding P(t) differences to be changed with the condition of independence and normality of ln P(t) differences.

263 Osbourn, M. F., 1959, „Brownian Motion in the Stock Market", *Operations Research*, vol. 7, pag. 145-173.

In this kind of conditions, for the stocks recording an acceptable liquidity (over 300-500 transactions per day) the central limit theorem assures a convergence of daily price changes (or daily returns) towards a normal distribution law. This conclusion is very important because it is validating the initial conditions of many fundamental models in finance, models using usually stock daily returns. Empirical proofs in assessing normality for price changes were offered beginning with the year 1950. Thus, [Kendall, 1953[264]] found that weekly changes of wheat price at the commodity exchange of Chicago and the prices of the most important stocks listed at the London Stock Exchange follows with a certain approximation a normal distribution law while [Moore, 1962[265]] reaches similar results in the case of changes of the price logarithm for a sample of stocks listed at New York Stock Exchange.

The stable paretian hypothesis

Starting with the year 1960, critical opinions related to the normality hypothesis advocated mainly through papers written by Bachelier and Osbourn found a strong argumentation in the person of Benoit Mandelbrot, professor at Harvard University. He considered that the previous empirical studies had unjustified amplified certain correlations between empirical distribution of stock price changes and the normal distribution neglecting in the same time some significant biases from normality. In this direction were brought in debate the empirical studies carried out by Kendall and Moore both of them concluding that the extreme tails of the empirical distributions are higher (they contain a bigger proportion from the total probability) compared with normal distributions. In other words, extreme variations, both the negative and the positives ones, are much more frequent than a normal distribution would imply. Conform to Mandelbrot's opinion these biases are sufficient in order to justify a new radical approach of the random walk theory for speculative financial assets. This new approach, known as the stable paretian hypothesis, is based mainly on two ideas: the variances of the empirical distributions are behaving as they were infinite respectively the empirical distributions are behaving like non-Gaussian members of a limit distributions family that is the stable paretian hypothesis.

From the beginning, the expression of radical changed approach is justified by the fact that the variance of the empirical distributions generally is assumed to be infinite, a hypothesis with very important consequences. Thus, from a purely statistical approach, if the variance associated with the distribution of populations of first order price differences ($P_{(t+1)}$-$P_{(t)}$) is infinite than the variance associated with samples from these populations represents in itself an insignificant measure of dispersion. Moreover, if the variance is infinite than merely all statistical methods which are starting from assumptions of a finite variance will be, in the best case, very weakened from the significance point of view and they might finalize in very misleading answers or results (a fundamental example is running regressions using the least squares method). In consequence, due to the fact that previous empirical researches regarding the speculative financial asset prices were founded on statistical techniques which consider a finite variance, the real value of many of them is thus untruthful if Mandelbrot's hypothesis were to be confirmed by empirical data.

For this purpose, we remind the fact that the first tests of the stable paretian hypothesis were made on a limited number of speculative price changes series and that each direct test on this unprocessed and unsmoothed data found a behavior which was predictable by the paretian hypothesis. Ulterior tests confirmed that in the case of speculative financial assets price changes (and we speak mainly about the stock and cotton prices) first order differences of prices' logarithm follow paretian distributions. A rigorous empirical research of the stable paretian hypothesis was made by Eugene Fama in his renowned article from 1965[266], the resulted conclusions stressing the following:

„[…] in particular, Mandelbrot's hypothesis asserts that the empirical distributions of price changes are fitting better into the category of stale paretian distributions with a characteristic exponent smaller than 2 compared with the normal distributions category (which are after all stable paretian distributions but with a characteristic exponent equal to 2). The conclusion of this paper is that the Mandelbrot hypothesis seems to be sustained by the empirical data. To this conclusion we have reached only after an extensive and complex testing was carried out."

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[264] Kendall, M. G., 1953, „The Analysis of Economic Time-Series", *Journal of the Royal Statistical Society*, vol. 96, pag. 11-25.
[265] Moore, A., 1962, „A Statistical Analysis of Common –Stock Prices", *University of Chicago*, PhD dissertation.
[266] Fama, E., 1965, „The Behavior of Stock Market Prices", *Journal of Business*, vol. 38, pag. 34-105.

Following these serious documentations of stock returns behavior we can speak generally about the stock exchange as a speculative market in the sense of the stable paretian hypothesis. Still, there are significant differences from a market to another and in many cases biases from normality are too insignificant in order to justify a radical change of approach. This radical change is less needed especially when the aggregating interval of price changes gets big enough, for example if we speak about weakly or monthly returns, cases in which the non normality hypothesis can be accepted in a comfortable way.

Still, the existence of too high frequencies in the case of extreme variations impose the consideration of the stable paretian hypothesis in order to be able to intervene on this kind of markets with a certain level of rationality and in the virtue of efficiency thus implied is necessary to known the statistical characteristics of the probability distributions from the stable paretian family and also their influence on stock returns' behavior.

The stable paretian distribution laws family

The starting point of the stable paretian hypothesis is represented by the works in mathematical analysis related to the notion of stable distribution. A certain distribution assumes a conservation of the distribution characteristics when identically distributed variable are summed up. This means that if two variables x and y are distributed according to a stable distribution then their sum x+y is identically distributed. Notable contributions to this field of stable distributions were made by the French mathematician Paul Levy which deducted the characteristic function associated to all stable distributions. The results of his research were materialized in his book "Theorie de l'addition des variables aleatoires" published in 1937, a point which marked the beginning of some future developments mainly in the field of probability theory and statistics.

Not without any interest is the fact that Benoit Mandelbrot, the enthusiastic advocate of the stable paretian hypothesis in the area of speculative markets, represents the continuation of a special series of doctoral advisors – PhD students. Thus, he was the student of Paul Levy at the Ecole Polytechnique of Paris, which was the student of Jacques Hadamard (accidentally Dreyfus brother in law) which was the student of Charles E. Picard, the last two being two of the greatest mathematicians of the last century.

As I have mentioned earlier, the characteristic function associated to all stable distributions was deducted

$$\log f(t) = \log \int_{-\infty}^{\infty} e^{iut} dP(\tilde{u} < u) = i\delta t - \gamma |t|^{\alpha} \left(1 + i\beta \frac{t}{|t|} tg \frac{\alpha\pi}{2}\right)$$

by Paul Levy in the specialized literature being used generally under its logarithm form:

From the above characteristic expression it cam be noticed that when defining stable paretian distribution four factors are needed ($\alpha,\beta,\delta,\gamma$) as opposed for example to the case of the normal distribution defined only by two, the mean and the standard deviation. We present the four parameters and their significance.

1. *Location parameter* (δ): if $\alpha>1$, it represents the expected value (mean) of the distribution.
2. *Scale parameter* (γ);
3. *Skewness index* (β): can take values between –1 and 1;
   - (a) if $\beta=0$ then the distribution is symmetrical;
   - (b) if $\beta>0$ then he distribution exhibits a positive asymmetry and the degree is increasing as β approaches 1;
   - (c) analog for $\beta<0$ mentioning that the asymmetry is negative.
4. *Characteristic exponent or stability parameter* ($0<\alpha\leq 2$).

From the above four parameters, the characteristic exponent α is the most important if we are considering comparing the Gaussian hypothesis with the stable paretian because α determines the height (the probability/frequency) of the extreme tails and can take any value within the (0,2] interval. The two hypothesis are identical only when $\alpha=2$[267]. If $\alpha\in(0,2)$ then the extreme tails of the paretian distribution are

[267] The logarithm of the characteristic function for the normal distribution is: $\log f(t) = i\mu t - \frac{\sigma^2}{2} t^2$, with $\alpha=2$, $\beta=0$, $\delta=\mu$, $\gamma=\sigma^2/2$.

higher than those of the normal distribution with a total probability increasing with α moving from 2 to 0. A very important aspect is that the variance exists and it is finite only in the case when $\alpha=2$, that is only for the normal law in all other cases being infinite. On the other hand, the expected value exist for any value of $\alpha>1$. In conclusion, the stable paretian hypothesis advocated by Mandelbrot says that for probability distribution associated to the subsequent price changes the characteristic exponent α is situated within the (1,2) interval and as a consequence these distributions have an expected value but their variance is infinite.

Because the conflict between the two hypothesis are concentrated especially near the α value, a rational choice between the two hypothesis at least at a theoretical level can be made only by estimating the true value of the α parameter. But this task is not an easy one because the explicit expressions of the densities for stable paretian laws are known only in three cases: the normal distribution ($\alpha=2$), the Cauchy distribution ($\alpha=1$, $\beta=0$) and the coin tossing experiment ($\alpha=1/2$, $\beta=1$, $\delta=0$, $\gamma=1$). Without the knowledge of any other density functions is very hard to introduce and prove different assumptions related to the behavior of α estimates in the case of samples. Still, significant empirical progresses were made in this matter in [Fama, 1965].

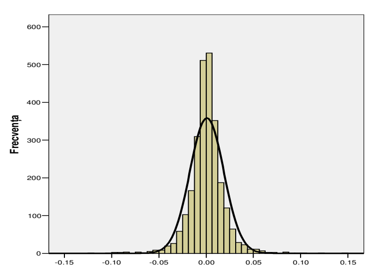
Financial implication of the stable paretian hypothesis at the Bucharest Stock Exchange

In this section e will graphically analyze the probability distribution associated with the logarithmic returns of the two main indexes at Bucharest Stock Exchange (BSE), that is BET and BET-FI as well as two intensely traded stocks, Banca Transilvania (TLV) and SIF Moldova (SIF2). In the same time the main statistical characteristics will be summarized in table 1. For each stock or index we will consider the relation between the empirical distribution and a normal distribution having as parameters the average and the standard deviation corresponding to the observed sample of returns. In the case of the BET index we used a sample of 2,571 daily logarithmic returns in the period 19.09.1997-7.02.2008, a graphical representation of the empirical probability distribution being in displayed in figure 1.

As it can be easily noticed, the empirical returns around the zero value are much more frequent than the normal distribution would imply, this one being marked with a continuing line in the chart resulting the pronounced leptokurtic character. A more detailed view, available in figure 2 reveals also a perspective upon the extreme returns, positive and negative, again much larger than the Gaussian hypothesis would imply. On the other hand, it seems that the asymmetry degree is not so high since the designed coefficient takes a small value of just 0.1 compared to the zero value (see table 1). Still, speaking about all the returns from the sample we can conclude that the distribution of the empirical returns is more peaked and presents extreme tails more preeminent than a normal distribution indicating a rejection of the Gaussian hypothesis and an acceptance of the stable paretian.

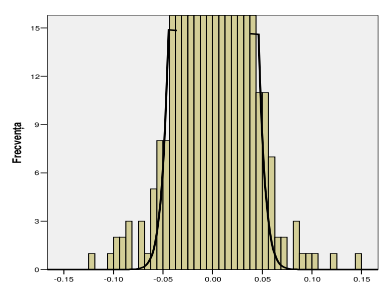
In a similar way, the above conclusions remain valid for the five financial companies index, BET-FI (sample formed by 1.778 returns, observed in the period 31.10.2000 and 7.02.2008), a higher volatility being noticed respectively a kurtosis coefficient more close to the normal distribution.

Figure 1: Histogram of the daily returns in the case of the BET index (1997-2008)



Source: Author's manipulations in SPSS.

Figure 2: Histogram of daily returns in the case of the BET index (1997-2008) (detail)



Source: Author's manipulations in SPSS.

Related to this observation, we consider that an intuitive aspect also exists for the relation between the volatility associated to a probability distribution and the high density of the returns around the zero value. For a stock or a market index which is little traded at large periods of time a liquidity problem arises caused mainly by a low interest from the part of the investors for the specific financial asset. In this kind of conditions, the daily average volatility in the case of a sample with an acceptable financial and statistical dimensions (recommendable over five years of trading) will be somehow low because of the days in which the financial asset does not record large price changes or, more sadly, when it is not traded. In such periods, the returns will be very close to the zero value leading to a leptokurtic distribution with high frequencies around the zero value.

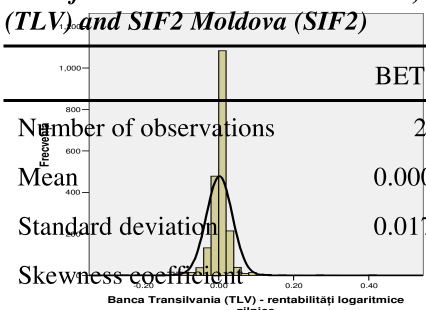
When we analyze the empirical distributions of the TLV and SIF2 stocks, we can notice in the first place the significance with whom the individual characteristics of those are exhibited (see table 1). Thus, both stocks exhibit a significant positive asymmetry of 2.59 and 1.47, because the two stocks have recorded high price growth rhythms in this period of time. On the other hand, the kurtosis coefficient records very high values of 45.29 and 38, because the null or very small returns were not compensated by some more volatile evolutions of other stocks like it was the case with two indexes previously analyzed. The two significant biases, related to the asymmetry and the kurtosis as well as an inspection of the associated charts are leading to a rejection of the Gaussian hypothesis. In the same time, it is also empirically verified that the indexes returns are approaching significantly closer to the Gaussian hypothesis than the returns of individual stocks.

Based on the previous exemplifications we can state some financial implications of the stable paretian hypothesis. Firstly, this hypothesis implies that on the speculative markets the equilibrium prices are formed as a consequence of much more extreme variations compared to the Gaussian hypothesis. This means that a paretian market is inherently riskier than a Gaussian one, aspect reflected by a variation interval associated to the expected value of the return higher respectively a bigger loss probability.

Table 1: Main characteristics of the empirical distributions of he daily returns in the case of the BET and BET-FI indexes, respectively the stocks Banca Transilvania (TLV) and SIF2 Moldova (SIF2)

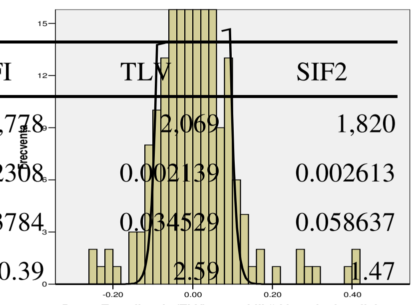
	BET	BET-FI	TLV	SIF2
Number of observations	2,570	1,778	2,069	1,820
Mean	0.000774	0.002308	0.002139	0.002613
Standard deviation	0.017909	0.023784	0.034529	0.058637
Skewness coefficient	0.10	0.39	2.59	1.47
Kurtosis coefficient	10.46	8.97	45.29	38.00

Figure 3: Histogram of daily returns in the case of Banca Transilvania



Source: Author's manipulations in SPSS.

Figure 4: Histogram of daily returns in the case of Banca Transilvania (details)



Source: Author's manipulations in SPSS..

Secondly, on a paretian market the speculators cannot assure themselves constantly against huge losses using instruments like the stop-loss orders and this is due mainly because while for a Gaussian market a huge variation is likely to be formed by many smaller changes on a paretian market it is very probable that the huge change to be the results of just a few large price changes[268]. This means that if the market will record a downfall in the stock prices, this correction will be very rapidly and not allowing for many intermediate prices necessary for the execution of the stop-loss orders. This aspect could be very easily noticed at Bucharest Stock Exchange (BSE) especially in January-February period of the years between 2004 and 2007 and of course 2008. When during a trading session at the stock exchange, many stocks

[268] The proof can be found in the following article: Darling, D., 1952, „The Influence of the Maximum Term in the Addition of Independent Random Variables", *Transactions of the American Mathematical Society*, vol. 15, pag. 95-107.

record price falls of more than 5% it is almost impossible for the investors to protect themselves through this stop-loss orders or different derivatives instruments.

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