

# VOLATILITY ESTIMATION BASED ON HETEROSKEDASTIC MODELS VS. HISTORICAL MODELS<sup>134</sup>

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*Abstract: Volatility represents the most common used method for risk estimation. The aim of this paper is to estimate volatility based on historical models and heteroskedastic models, GARCH (!,!), in order to identify the crisis moment within American capital market. The database used consists in daily observations about Dow Jones index, spanned within 1928 – 2002.*

*Keywords: volatility, heteroskedasticity, GARCH, capital market*

## **Introduction**

Following the seemingly work of Bachelier (1900) and Markowitz (1952), volatility represents the most common used method for risk estimation among a long list of other methods (see in that sense, Pedersen and Satchell, 1998). The main reasons which support the previous affirmation are: (i) the fact that each rational investor is characterized by a squared utility function, and (ii) financial assets' returns normal distribution hypothesis. But, both assumptions were not entirely demonstrated by empirical studies and tests.

Using a second order polynomial utility function proves to be inconsistent with the fact that averse risk investors could seldom decide to invest in risky financial assets (Kahneman and Tversky, 1979), or with a cautious investors' behavior (Kimball, 1993).

The hypothesis of normal distribution probability of financial assets is not confirmed by empirical studies (see in that sense, Fama, 1965 and Mandelbrot, 1997). In fact, returns distribution is asymmetric and leptokurtic (Christie and Andrew, 1982). Using the leverage effect, active and passive portfolio management strategies and derivatives go to convex profit functions (Bookstaber and Clarke, 1981), while credit and liquidity risk are the main source of potential large losses on financial markets. Long run investment strategies and returns heteroskedasticity are the causes of financial assets asymmetric and leptokurtic probability distribution (Fama, 1996; Bollerslev, 1986).

Consequently, using volatility as good estimator for risk is inconsistent with investors' informational asymmetry or the statistics of returns distribution. Even if volatility represents a friendly method, it has some drawbacks. For instance, volatility is an unobservable element, which makes its estimation very difficult. Based on stochastic models, there were explained dynamic characteristics of volatility, so it could be measured using regular observations on financial assets' prices. It is a stochastic volatility, which is not constant over time and depends on the average conditioned by past shocks. The estimation of such process

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is difficult, and the estimators could be distorted (Barndorff-Nielsen and Shephard, 2002; Bollerslev and Zhou, 2002).

The aim of this study is to estimate volatility using historical methods and heteroskedastic methods, GARCH (1,1).

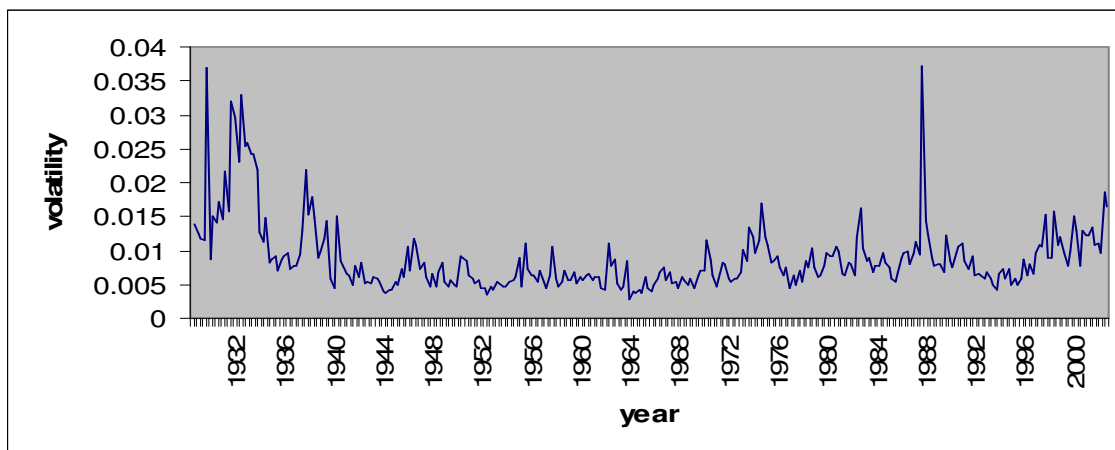
**Database**

The database used is represented by closing quotations of Dow Jones index within October 1928 – December 2002.

**Empirical results**

Historical volatility was estimated as standard deviation at the end of each quarter based on daily rates of returns, calculated as natural logarithm of closing quotations of Dow Jones index.

The estimated quarterly volatility based on historical methods is presented in Figure 1.



**Figure 1 Historical estimated quarterly volatility**

In order to estimate volatility based on heteroskedastic methods, it was used a GARCH (1,1) model, similar to one proposed by Bollerslev for obtaining a maximum likelihood. Model’s equations are the following:

$$r_t = \mu + \rho \cdot r_{t-1} + \varepsilon_t \tag{1}$$

$$\sigma_t = a_0 + a_1 \cdot \sigma_{t-1} + b_1 + \varepsilon_{t-1}^2 \tag{2}$$

where:

$r_t$  : rate of return calculated related to return at t-1;

$\sigma_t$  : estimated volatility depending on volatility at t-1 and residuals at t-1;

$\mu, \rho, a_0, a_1$  and  $b_1$  : coefficients.

In order to obtain relevant results, residual term,  $\varepsilon = N(0,\sigma_t)$ , has to follow normal distribution of zero average and variance equal to volatility.

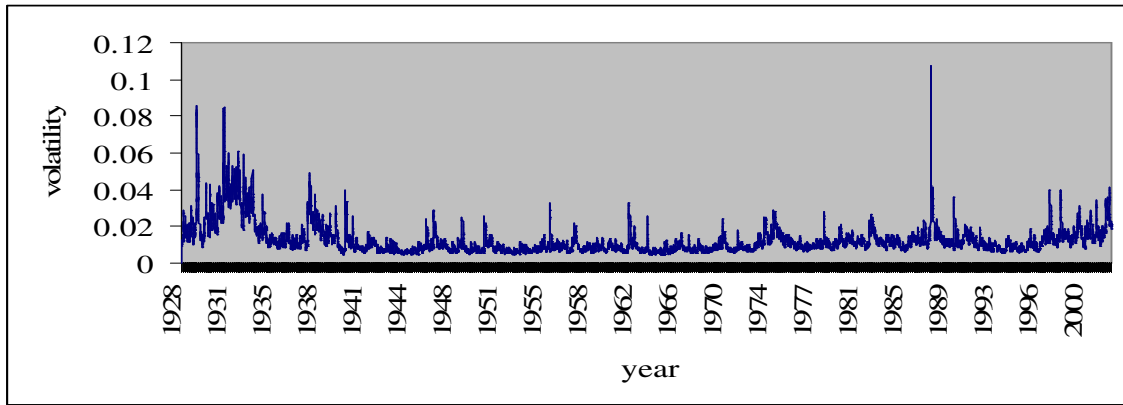
GARCH (1,1) estimations go to a maximum likelihood of 65691.3270, and estimated parameters are presented in the table below:

	$\mu$	$\rho$	$a_0$	$a_1$	$b_1$
<b>Estimated value</b>	0.0004	-7.7400	-0.0009	12.0740	3.5962
<b><math>\sigma^2</math> parameters</b>	0.0001	0.0076	0.0000	0.0041	0.0038
<b>T-statistic</b>	6.8156	14.9179	8.7173	221.3062	21.1474

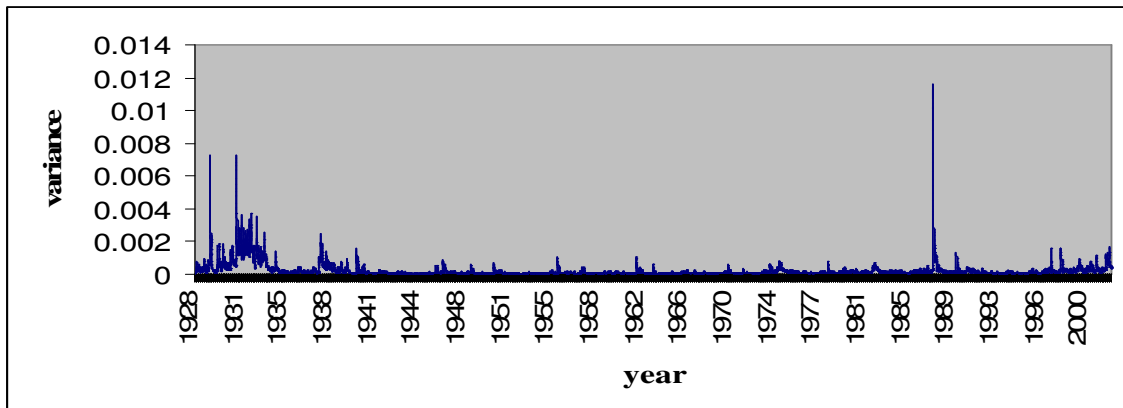
**Table 1. Estimated GARCH**

The estimated results show the coefficients significance.

The estimated volatility and variance is presented in Figure 2 and 3.

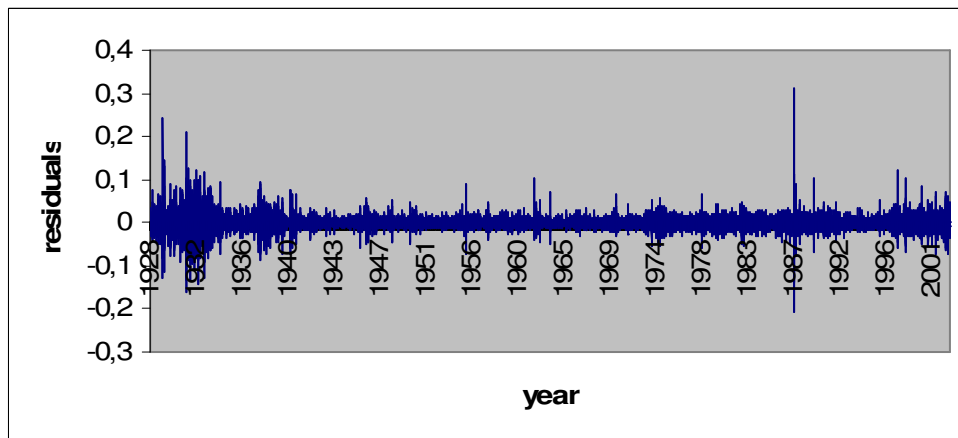


*Figure 2. Estimated daily volatility based on heteroskedastic models*

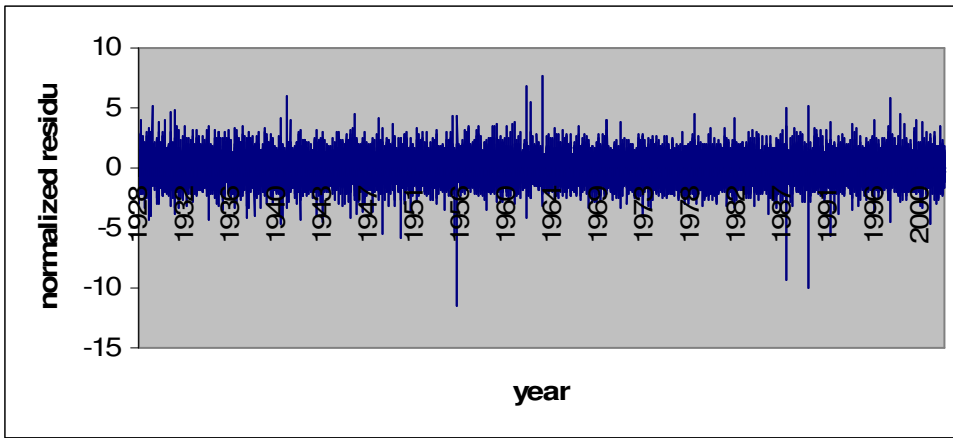


*Figure 3. Estimated daily variance based on heteroskedastic models*

The estimated residuals are presented in Figure 4 and 5.

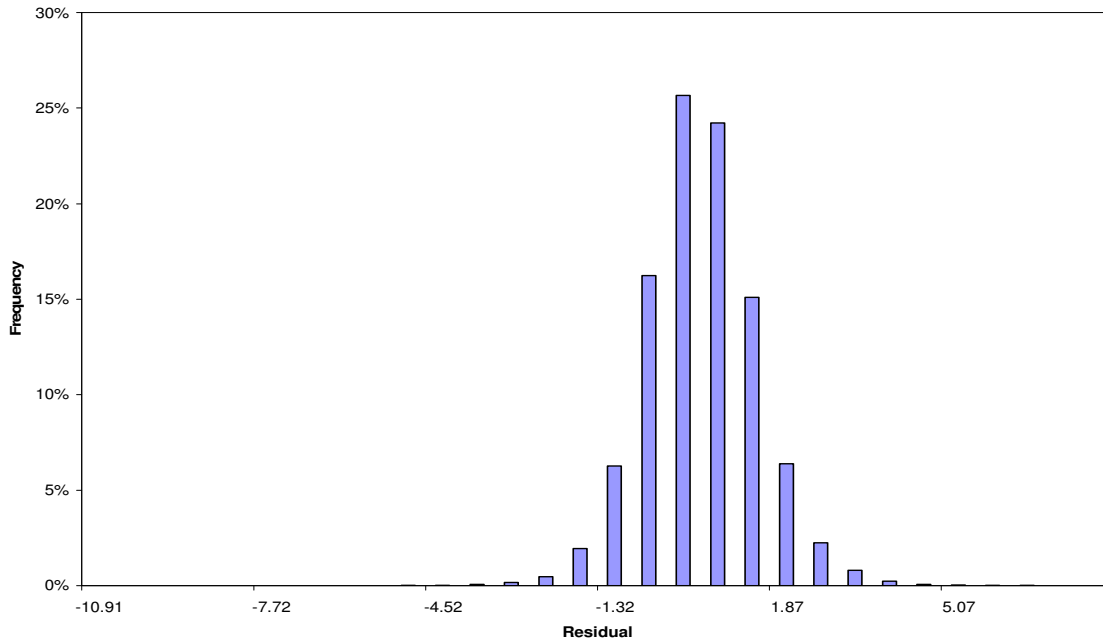


*Figure 4. Return estimated residuals*

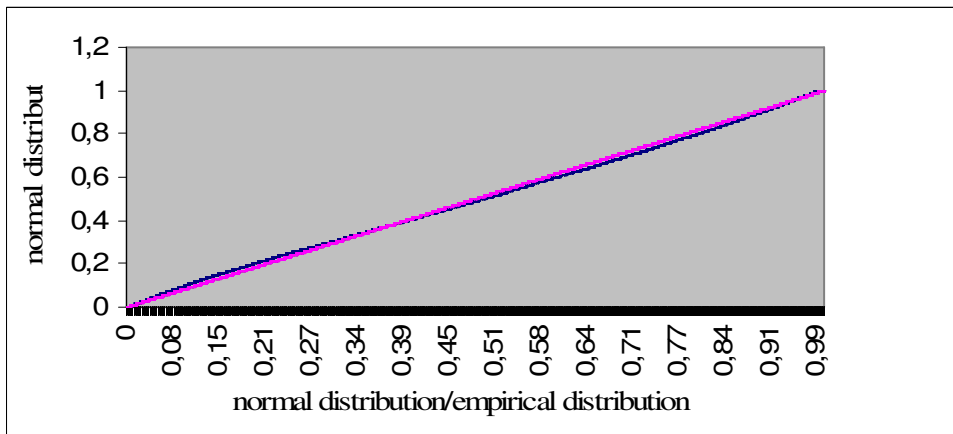


*Figure 5. Normalized residuals*

Normal distribution of residuals is presented in Figure 6 and 7.



*Figure 6. Residuals empirical distribution*



### *Figure 7. Residuals Henry line*

#### **Concluding remarks**

The estimated volatility based on historical methods and on GARCH (1,1) model reveal the crisis from American capital market. It could be noticed some crisis moments, such as the peak within 1928 – 1933 (characterized by large volatility and duration), and within 1986 - 1988 (characterized by large volatility and small duration, only one month, at the end of 1987). Both moments are revealed by the historical estimated volatility and by GARCH estimated volatility, but the difference appears related to their size. There were, also, identified some other distortions on US capital market at 1938, 1941, 1946, 1950, 1957, 1960, 1962, 1966, 1970, 1973-1975, 1978 – 1981, 1982 – 1984, 1990. After 1997, it is noticed an increasing volatility for 1997, 1998, 2000, 2001 and 2002.

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