AN ANALYSIS OF EQUITY IN INSURANCE. THE MATHEMATICAL APPROACH OF RISK OF RUIN FOR INSURERS

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Abstract: The goal of the present paper is a short analysis for the insurers' foreign equity in Romania and the development of a mathematical approach for the chronological evolution of the study regarding the insurers' equity from the point of view of assessing the insolvency probabilities and the risk provision so the estimating insolvency risk will not over overcome an accepted value.

Key words: Insurer, broker, adjusting coefficient, overcharging factor, probability of ruin, compound Poisson repartition.

Introduction

Since 1989, Romania faced a development of private property and a concentration of indigene capital, facts that determined continuous growth of the insurance market. In a first phase, the permissible legal framework (Law no. 47/1991) and the lack of the Romanian capital but also the attractiveness of the Romanian market were the main factors for the market penetration of the foreign capital on the Romanian insurance market. The same phenomenon took place in the insurance mediation sector.

The number of insurers reached a normal evolution, with a spectacular growth at the beginning of the insurance market, followed by a decrease according to the concentration and centralization of the capital, as shown below: (P is the percent of foreign capital to equity of the insurers).

Year	Number of insurers	P %
1997	47	-
1998	64	-
1999	72	62,11
2000	73	75,80
2001	47	61,70
2002	48	50,20
2003	44	59,21
2004	45	48,00
2005	43	50,00
2006	41	53,10

Source: - OSAAR Reports, Bucharest, years 1997-1999;

- ISC Reports, Bucharest, years 2000-2006;

As it can be seen, a significant share of equity is represented by the foreign capital, mainly Austrian or German. Since year 2007, it is evident a foreign capital domination due to the fact that the first 6 insurers have a market share of more than 70%. Since year 2000, there are changes in the sales management for the insurance products, so the insurance agent is replaced by the insurance broker. The law 32/2000 sets a clearer view on mediation sector, especially for brokers. This is supposed to be a legal person, to prove cash equity, to have valid third party liability insurance, and the only subject of activity – the insurance mediation, to have qualified human resources.

The evolution for the number of brokers is shown below:

Year	1999	2000	2001	2002	2003	2004	2005	2006
Number of brokers	749	817	198	150	204	266	317	344

Source: O.S.A.A.R. and ISC Reports, Bucharest, years 1999-2006.

It is important to assess the risk that an insurance company faces bankruptcy. In order to explain this, we shall provide a mathematical model regarding the insurer's capital and we shall determine the ruin probability, this means the probability that the insurer has no more resources to pay the indemnities (obviously this isn't a de facto ruin, but an attention for the financial management).

2. The modeling of risk of ruin

In order to study the variation of insurer's equity, we shall note with $U(t)$ the insurer's equity at moment t, with u the initial capital, with $S(t)$ the total claim demand (more precisely the amount of total claim demand) occurred till the moment t, with m the number of insurance policies, with $N_k(t)$ the number of individual claim demands till the moment t for a given policy type k, with Y_i^k the individual demand number i for the insurance type k and with $S_k(t)$ the claim demand occurred till the moment t for a given policy type k.

We have:

$$S(t)=\sum_{k=1}^{m} S_k(t) \text{ and } S_k(t)=\sum_{i=1}^{N_k(t)} Y_i^k .$$

Additionally, we presume the following hypothesis to be fulfilled:

1. there are no other expenses than the paid indemnities and no other incomes than the cashed premiums.
2. the unit income is c.
3. the individual claim demands for every policy type are independent identically distributed random variables. The individual claim demands for every insurance type are independent and identically distributed variables
4. the stochastic processes $(N_k(t))_t$, $k=\overline{1,m}$, for the number of demands are Poisson processes of parameter λ_k. It results that process $\{S(t)\}_t$ is a compound Poisson process composed of parameter $\lambda=\sum_{k=1}^{m}\lambda_k$, so we have $S(t)=\sum_{i=1}^{N(t)} X_i$, where random variables X_i are independent and identically distributed, having the moment of order k noted with m_k, $m_k=M\left(X^k\right)$. The variables X_i equalize mathematically the individual real claim demands Y_i^k, but there are not identically distributed. In these hypothesis, we have $M(S(t)) = \lambda\cdot t\cdot m_1$ and $D^2(S(t)) = \lambda\cdot t\cdot m_2$.
5. The insurance premiums are determined with respect to the medium value principle, with an overcharging relative confidence θ:

$$\text{k}\, c=(1+\theta)\cdot m_1\cdot M(N(1)) .$$

We have: $U(t) = u + c \cdot t - S(t)$ (1).

We shall name ruin the situation this capital is negative and we shall consider the moment of ruin at the moment this happen for the first time. So we are dealing with the problem of ruin for a given, undetermined period of time, called finite horizon. So:

$T = \inf\{t > 0 \mid U(t) < 0\}$ is the moment of ruin.

In this context, the ruin probability (noted with Ψ) is the probability that the moment of ruin is finite. From the point of view of initial capital *u*, the overcharging factor θ, the claim files rate λ and the medium individual claim demand m_1, the function is: $\Psi(u,\theta,\lambda,m_1) = P(T < \infty)$.

The equation (in r) strictly positive solution: $\lambda + c \cdot r = \lambda \cdot M\left(e^{r \cdot X}\right)$ is called adjustment coefficient. This coefficient (when does exist) shall be noted as *R*.

Considering an existing adjustment coefficient *R*, we have:

$$\psi = \frac{e^{-R \cdot u}}{M\left(e^{-R \cdot U(T)} \mid T < \infty\right)}.$$

If the adjustment coefficient *R* does exist, then $\Psi(u) \le e^{-R \cdot u}$ (this is known as the Cramer inequality).

This is determined by the fact that for a finite *T* (*T* < ∞), we have *U*(*T*) < 0, so $M(e^{-R \cdot U(T)} \mid T < \infty) > 1$.

Generally speaking, because the adjustment coefficient is difficult to compute, we seek for a convenient interval, whose margins are used in the Cramer inequality. Because $M\left(e^{R \cdot X}\right) > 1 + R \cdot m_1 + \frac{R^2}{2} \cdot m_2$ we obtain: $R < \frac{2 \cdot \theta \cdot m_1}{m_2}$. Considering the function $\omega(r) = \lambda \cdot (m_X(r) - 1) - c \cdot r$ which is strictly convex and due to the fact that net profit condition is fulfilled ($\omega'(0) = \lambda \cdot m_1 - c < 0$) , the results are $\omega(R) = \omega(0) + \int_0^R \omega'(s) \cdot ds > \lambda \cdot m_1 \cdot R - c \cdot R + \lambda \cdot m_2 \cdot \frac{R^2}{2}$ and $R < \frac{2 \cdot c - 2 \cdot \lambda \cdot m_1}{\lambda \cdot m_2}$.

If the individual demands have an exponential repartition of parameter α (so $m_1 = \frac{1}{\alpha}$), because the medium income per unit time is $c = (1+\theta) \cdot \frac{\lambda}{\alpha}$, we find for the ruin probability the expression:

$$\psi(u,\theta,\lambda,\alpha) = \frac{\lambda}{\alpha \cdot c} \cdot e^{-\left(\alpha - \frac{\lambda}{c}\right) \cdot u} = \frac{1}{1+\theta} \cdot e^{-\frac{\alpha \cdot \theta \cdot u}{1+\theta}} \qquad (2).$$

Considering the initial capital as a β number of medium individual claim demands, we obtain:

$$\psi(\beta,\theta) = \frac{1}{1+\theta} \cdot e^{-\frac{\theta \cdot \beta}{1+\theta}} \qquad (3).$$

We could improve this model by setting the problem to compute a risk provision (*V*), in order to define the probability that paid indemnities should not overcome the incomes plus this provision, under an accepted value *p*. If the medium number of claim demands (in our case $n = \lambda \cdot t$) is big enough, we can use the central limit theorem (TLC) and approximate $S(t)$ with a normal variable of mean $\lambda \cdot t \cdot m_1$ and dispersion $\lambda \cdot t \cdot m_2$. So we want to determine *V* in order that $P\left(S(t) \ge u + c \cdot t + V\right) \le p$. Using TLC we obtain $V \geq z_{1-p} \cdot \sqrt{\lambda \cdot t \cdot m_2} - u - \theta \cdot \lambda \cdot t \cdot m_1$, where z_{1-p} is the quartile of order $1-p$ of the normal repartition. If we choose the minimum risk reserve in order not to block higher financial resources, we have:

$$V = z_{1-p} \cdot \sqrt{\lambda \cdot t \cdot m_2} - u - \theta \cdot \lambda \cdot t \cdot m_1 \qquad (4)$$

In the hypothesis that individual claim demand follows an exponential repartition of parameter α, so $m_1 = \frac{1}{\alpha}$, $m_2 = \frac{1}{\alpha^2}$, we have:

$$V = z_{1-p} \cdot \frac{\sqrt{\lambda \cdot t}}{\alpha} - u - \frac{\theta \cdot \lambda \cdot t}{\alpha} \qquad (5)$$

Choosing an initial equity to represent a number of individual medium claim demands (β), we obtain:

$$V = \frac{z_{1-p} \cdot \sqrt{\lambda \cdot t} - \beta - \theta \cdot \lambda \cdot t}{\alpha} \qquad (6)$$

3. Numerical results

From relation (3) we have determined the values for ruin probabilities for a few values of premiums overcharging factor θ and parameter β, that provide the number of medium individual demands to cover the initial equity.

The results are listed in the table below:

$\theta \backslash \beta$	5	10	20	30	50	60	100
0.1	0.5770	0.3663	0.1476	0.05945	0.00965	0.00428	0.0001
0.2	0.3622	0.1574	0.0297	0.0056	0.0002	$4.5 \cdot 10^{-5}$	$4.8 \cdot 10^{-8}$
0.4	0.1712	0.0410	0.0024	0.00014	$4.5 \cdot 10^{-7}$	$3.6 \cdot 10^{-8}$	$2.8 \cdot 10^{-13}$
0.6	0.0958	0.0147	0.0003	$8.1 \cdot 10^{-6}$	$4.5 \cdot 10^{-9}$	$1.7 \cdot 10^{-10}$	$3.2 \cdot 10^{-17}$

Obviously, the higher the initial equity and the bigger the overcharging premiums are, the smaller the ruin probability is. The diminution of risk of ruin realizes exponentially compared to the growth of those two factors. The increasing overcharging factor is limited by the necessity to maintain an adequate premium level for the competitiveness of the insurance company on market. We also emphasize this model is correlating to strong the incomes and the indemnities, the medium income per unit time being proportional to the medium number of individual claim demands multiplied by the value of medium individual claim demand.

This is due to the principle of medium value for the premiums computation. We consider the use of medium annual claim index instead of claim demand in order to obtain a more realistic model.

Using relation (6) we obtained the following results:

- for $p = 0.005, \alpha = 0.001, \lambda = 1, \ \beta = 2$, values of t and β, the risk reserve V (u.m.) is provided in the following tabel:

$t \backslash \theta$	0.1	0.2	0.5
1	480	380	80
5	3269	2769	1269

10	5159	4159	1159

– for $p = 0.001, \alpha = 0.01, \lambda = 5, \ \theta = 0.1$, values of *t* and β, the risk reserve *V* (u.m.) is provided in the following tabel:

$t \setminus \beta$	5	10	20
100	1590	1090	90
400	1680	1180	180

We underline that the negative values for the relations (4)-(6) mean a risk reserve equal to zero. We also ascertain the risk reserve to be proportional to the claim medium individual demand, decreasing when the initial equity covers many claim individual demands and decreasing when the requested margin for the insolvency risk decreases.

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